SHIKSHA CLASSES

Subject : Geometry | **Answer Paper** | **Marks : 20**

Class : X | **6. Trigonometry**

Q.1: A) Choose the correct alternative from the following questions. **2**

1) $1+\cot^2\theta = ?$

Ans: b) $\text{cosec}^2\theta$

2) **When we see at a higher level, from the horizontal line, angled formed is ____**

Ans: a) Angle of elevation

B) Solve the following questions. **1**

1) **If $3\sin\theta - 4\cos\theta = 0$, find the value of θ**

Ans: $3\sin\theta - 4\cos\theta = 0$ [Given]

$3\sin\theta = 4\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\tan\theta = \frac{4}{3}$

Q.2: A) Attempt any ONE of the following. **2**

1) **If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$ by using identity.**

Ans: $\tan^2\theta = \boxed{\sec^2\theta - 1}$ ...... (Identity)

$= \left(\frac{25}{7}\right)^2 - 1$

$= \boxed{\frac{625}{49}} - 1 = \frac{625-49}{49} = \boxed{\frac{576}{49}}$

$\tan\theta = \boxed{\sqrt{\frac{576}{49}} = \frac{24}{7}}$

2) **If $\cos\theta = \frac{5}{13}$, find the value of $\text{cosec}\theta$ by using identity.**

Ans: $\cos\theta = \frac{5}{13}$ (Given)

$\sin^2\theta = 1 - \cos^2\theta$ [Identify]

$= 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{169-25}{169} = \frac{144}{169}$

$\therefore \sin\theta = \frac{12}{13}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$ [Identity]

$= \frac{13}{12}$

B) Attempt any ONE of the following. **2**

1) **If $\cot\theta = \frac{15}{8}$, find the value of $\sin\theta$.**

Ans: $\cot\theta = \frac{15}{8}$ [Given]

$\text{cosec}^2\theta = 1 + \cot^2\theta$ [Identity]

$= 1 + \left(\frac{15}{8}\right)^2$

$= 1 + \frac{225}{64}$

$= \frac{64+225}{64} = \frac{289}{64}$

$\text{cosec}\theta = \frac{17}{8}$

$\sin\theta = \frac{1}{\text{cosec}\theta} = \frac{8}{17}$

2) **Prove that $\cos^2\theta(1+\tan^2\theta) = 1$**

Ans: L.H.S. $= \cos^2\theta(1+\tan^2\theta)$

$= \cos^2\theta\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)$

$= \cancel{\cos^2}\theta\left(\dfrac{\cos^2\theta+\sin^2\theta}{\cancel{\cos^2}\theta}\right)$

$=\cos^2\theta+\sin^2\theta$

$= 1$ ..... ($\therefore$ fundamental Identity I)

= R. H. S.

$\boxed{\cos^2\theta\left(1+\tan^2\theta\right)=1}$

Q.3: A) Attempt any ONE of the following. 3

1) If $5\sec\theta-12\operatorname{cosec}\theta=0$, find the values of $\sec\theta, \cos\theta$ and $\sin\theta$.

Ans: $5\sec\theta-12\operatorname{cosec}\theta=0$ ...... (Given)

$5\sec\theta=\boxed{12\ \operatorname{cosec}\theta}$

$\therefore \dfrac{\sec\theta}{\operatorname{cosec}\theta}=\dfrac{\boxed{12}}{\boxed{5}}$

$\therefore \dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}=\dfrac{\boxed{12}}{\boxed{5}}, \quad \dfrac{\sin\theta}{\cos\theta}=\dfrac{12}{5}$

$\tan\theta=\dfrac{\boxed{12}}{\boxed{5}}$

$\sec^2\theta=\boxed{1+\tan^2\theta}$ ---(Identity)

$=1+\dfrac{144}{25}$

$=\boxed{\dfrac{169}{25}}$

$\sec\theta=\dfrac{13}{5}$

$\cos\theta=\dfrac{1}{\boxed{\sec\theta}}=\dfrac{5}{13}$

$\sin^2=1-\boxed{\cos^2\theta}$ ---(Identity)

$=1-\left(\dfrac{5}{13}\right)^2=1-\dfrac{25}{169}=\dfrac{144}{169}$

$\therefore \sin\theta=\dfrac{\boxed{12}}{\boxed{13}}$

2) If $\tan\theta+\dfrac{1}{\tan\theta}=2$, then show that $\tan^2\theta+\dfrac{1}{\tan^2\theta}=2$,

Ans: $\tan\theta+\dfrac{1}{\tan\theta}=2$ (Given)

$\left[\tan\theta+\dfrac{1}{\tan\theta}\right]^2=2^2$

$\tan^2\theta+2\times\dfrac{1}{\tan\theta}\times\tan\theta+\dfrac{1}{\tan^2\theta}=4$

$\left[(a+b)^2=a^2+2ab+b^2\right]$

$\therefore \tan^2\theta+\dfrac{1}{\tan^2\theta}=4-2$

$\therefore \boxed{\tan^2\theta+\dfrac{1}{\tan^2\theta}=2}$

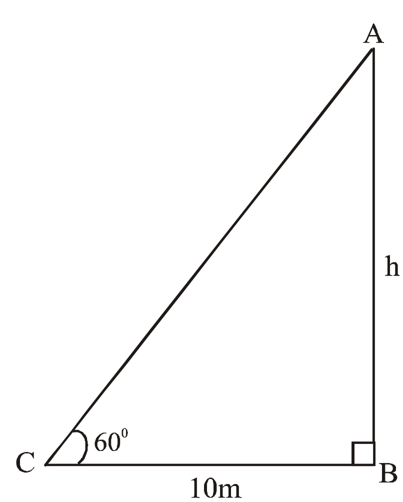
B) Attempt any ONE of the following. 3

1) An observer at a distance of 10m from a tree looks at the top of the tree, the angle of elevation is 60^0. what is the height of the tree? $\left(\sqrt{3}=.73\right)$.

Ans: In figure, $AB=h=$ height of tree

$BC=10\text{m}$, Distance of the observer from the tree Angle of elevation $(\theta)=\angle BCA=60^0$

from figure, $\tan\theta=\dfrac{AB}{BC}$ ________ I



$\tan 60^0 = \sqrt{3}$ ______ II

$\therefore \frac{AB}{BC} = \sqrt{3}$ [From I and II]

$\therefore AB = BC\sqrt{3} = 10\sqrt{3}$

$= 10\sqrt{3} = 10 \times 1.73$

$= 17.3$m

$\therefore$ height of the tree is 17.3m.

2) Two buildings are facing each other on a road of width 12 metre. From the top of the first building which is 10 metre high, the angle of elevation of the top of the second is found to be 60^0 what is the height of the second building?

Ans: Let AB and CD represents two buildings.
AB = 10m, BD is the width of the road.
BD = 12m

$\angle EAC = 60^0$ (Angle of elevation)

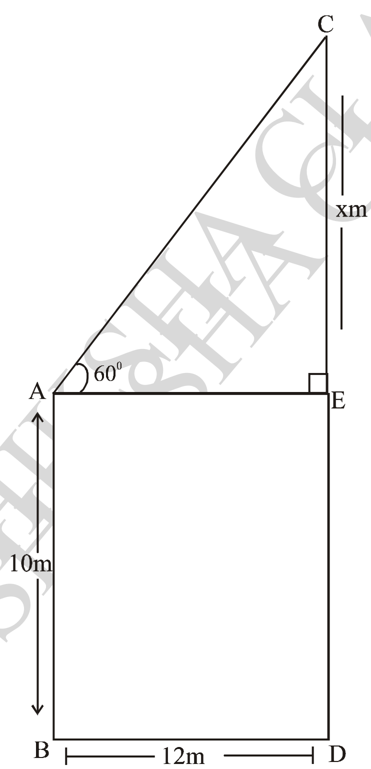
□ ABDE is a rectangle

$\therefore$ AE = BD = 12m ______ I

and AB = ED = 10m ______ II

Let CE be x m

In right angle ΔAEC,



$\tan \angle EAC = \tan 60^0 = \frac{CE}{AE}$

$\sqrt{3} = \frac{x}{12}$ [from I]

$x = 12\sqrt{3}$ m

$\therefore CE = 12\sqrt{3}$ m ______ III

$CD = CE + ED \quad \therefore [C - E - D]$

$= (10 + 12\sqrt{3})$m

Q.4: Attempt any one of the following questions. **4**

1) Show that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$

Ans: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \left(\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}\right) \times \left(\frac{\sin\theta + \cos\theta + 1}{\sin\theta + \cos\theta + 1}\right)$

$= \left(\frac{\sin\theta + 1 - \cos\theta}{\sin\theta + \cos\theta - 1}\right) \times \left(\frac{\sin\theta + 1 + \cos\theta}{\sin\theta + \cos\theta + 1}\right)$

$= \frac{(\sin\theta + 1)^2 - \cos^2\theta}{(\sin\theta + \cos\theta)^2 - 1^2}$

$= \frac{\sin^2\theta + 1 + 2\sin\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$

since, $\sin^2\theta + \cos^2\theta = 1$

Therefore,

$= \frac{1 - \cos^2\theta + 1 + 2\sin\theta - \cos^2\theta}{1 + 2\sin\theta\cos\theta - 1}$

$= \frac{2 - 2\cos^2\theta + 2\sin\theta}{2\sin\theta\cos\theta}$

$= \frac{\sin\theta + 1}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

$= (\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

We know that,

$$\sec^2\theta - \tan^2\theta = 1$$

Therefore

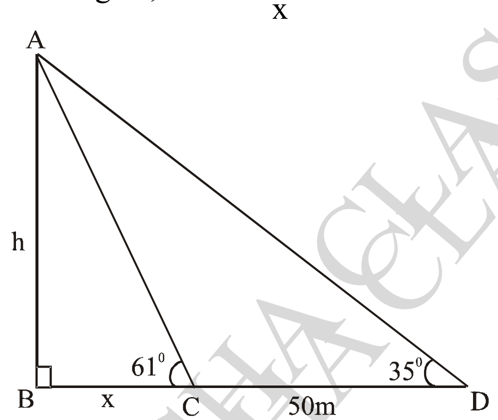
$$= \frac{1}{\sec\theta - \tan\theta}$$

Hence, it proved.

2) To find the width of the river, a man observes the top of a tower on the opposite bank making an angle of elevation of 61^0 When he moves 50m backward from bank and observes the same top of the tower, his line of vision makes an angle of elevation of 35^0. Find the height of the tower and width of the river. $\left(\tan 61^0 = 1.8, \tan 35^0 = 0.7\right)$

Ans: Seg AB shows the tower on the opposite bank 'A' is the top of the tower and seg BC shows the width of the river. Let 'h' be the height of the tower and 'x' be the width of the river.

From figure, $\tan 61^0 = \frac{h}{x}$



$$\therefore 1.8 = \frac{h}{x}$$

$$h = 1.8 \times x$$

10h = 18x ________ I (multiplying by 10)

In right angled ΔABD

$$\tan 35 = \frac{h}{x+50}$$

$$0.7 = \frac{h}{x+50}$$

$$h = 0.7(x+50)$$

$\therefore$ 10h = 7 (x + 50) ____ II

from equation I and II

18x = 7 (x + 50)

$\therefore$ 18x = 7x + 350

11x = 350

$$x = \frac{350}{11} = 31.82$$

Now, h = 1.8x 31.82 = 57.28m

$\therefore$ width of the river = 31.84m

and height of tower = 57.28m

Q.5: Attempt any ONE of the following. 3

1) Prove $\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x$

Ans: L. H.S. = $\sec^6 x - \tan^6 x$

We have,

$$sex^2 x - \tan^2 x = 1$$

Cubing on both sides, we get

$$(\sec^2 x - \tan^2 x)^3 = 1^3$$

$$\Rightarrow (\sec^2 x)^3 - (\tan^2 x)^3 - 3 \times \sec^2 x \times$$

$$\tan^2 x \times (\sec^2 x - \tan^2 x) = 1$$

$$\left[(a-b)^3 = a^3 - b^3 - 3ab(a-b)\right]$$

$$\Rightarrow \sec^6 x - \tan^6 x - 3\sec^2 x \tan^2 x = 1$$

$$\Rightarrow \sec^6 x - \tan^2 x = 1 + 3\sec^2 x \tan^2 x$$

2) Show that $\sec^4 A (1-\sin^4 A) - 2\tan^2 A = 1$

Ans: To prove

$\sec^4 A (1-\sin^4 A) - 2\tan^2 A = 1$

$$\sec^4 A - \sec^4 A \, x \sin^4 A - 2\tan^2 A = 1$$

$$\sec^4 A - \tan^4 A - 2\tan^2 A = 1$$

$$[\tan\theta = \sec\theta \times \sin\theta]$$

$$\left(\sec^2 A\right)^2 - \tan^4 A - 2\tan^2 A = 1$$

$$\left(1 + \tan^2 A\right)^2 - \tan^4 A - 2\tan^2 A = 1$$

$$\left[\sec^2\theta = 1 + \tan^2\theta\right]$$

$$1 + 2\tan^2 A + \tan^4 A - \tan^4 A - 2\tan^2 A = 1$$

$$\left[(a+b)^2 = a^2 + 2ab + b^2\right]$$

L. H. S. = R. H. S.

$\therefore$ $\sec^4 A (1-\sin^4 A) - 2\tan^2 A = 1$

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