SHIKSHA CLASSES

Sub. : Maths **Answer Paper** **Total Marks : 30**

Std. X (CBSE) **5 : Arithmetic Progression**

Section : A (Each 1 Mark)

Multiple choice Questions (MCQs).

Q.1 : In an Arithmetic Progression, if a = 28, d = -4, n = 7, then a_n is:

Ans : a) 4

Q.2 : 11th term of the A.P. –3, $-\frac{1}{2}$, 2 …. is

Ans : b) 22

Q.3 : The missing terms in AP: __, 13, __, 3 are:

Ans : c) 18 and 8

Q.4 : Which term of the A.P. 3, 8, 13, 18, … is 78?

Ans : d) 16th

Q.5 : The first four terms of an AP, whose first term is –2 and the common difference is –2, are

Ans : c) – 2, – 4, – 6, – 8

Q.6 : If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

Ans : c) 25

Q.7 : The middle most term (s) of the AP:–11, –7, –3, ..., 49 is:

Ans : c) 17, 21

Q.8 : If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be

Ans : d) 0

Q.9 : Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is

Ans : c) 7

For question number 10 to 11 two statements are given one labeled Assertion and other labeled Reason select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below

Q.10 : Assertion : If S_n is the sum of the first n terms of an A.P., then its nth term a_n is given by $a_n = S_n - S_{n-1}$.

Reason : The 10th term of the A.P. 5, 8, 11, 14, ………………. is 35.

Ans : c) If Assertion is correct but Reason is incorrect.

Q.11 : Assertion : The sum of the series with the nth term, $t_n = (9 - 5n)$ is 465, when no. of terms n = 15.

Reason : Given series is in A.P. and sum of n terms of an A.P. is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Ans : d) If Assertion is incorrect but Reason is correct.

Section : B (Each 2 Marks)

Q.12 : The sum of 5^{th} and 9^{th} terms of an AP is 72 and the sum of 7^{th} and 12^{th} terms is 97, find AP.

Ans : Let a be the first term and d be the common difference of the AP.

It is given that, $a_5 + a_9 = 72$ and $a_7 + a_{12} = 97$

$\Rightarrow a + 4d + a + 8d = 72$ and

$a + 6d + a + 11d = 97$

Thus, we have,

$2a + 12d = 72$ .... (i)

$2a + 17d = 97$ .... (ii)

$-\quad -\quad -$ Subtracting

$-5d = -25$

$\Rightarrow d = 5$

Putting d = 5 in (i) we get,

$2a + 12 \times 5 = 72$

$\Rightarrow 2a + 60 = 72$

$\Rightarrow 2a = 12$

$\Rightarrow a = 6$

Thus, a = 6, d = 5

Hence, the AP is 6, 11, 16, 21, 26, ...

Q.13 : How many numbers of two digits are divisible by 7 ?

Ans : As we know that 14 is the first two digits number divisible by 7 and 98 is the last two digits number divisible by 7.

Thus, we have to determine the number of terms in the sequence

14, 21, 28, ..., 91, 98

Clearly, it is an AP with first term = 14 and common difference = 7

i.e. a = 14 , d = 7

Let there be n terms in this AP, then,

n^{th} term = 98

$\Rightarrow a + (n-1)d = 98$

$\Rightarrow 14 + (n-1)\,7 = 98$

$\Rightarrow 7(n-1) = 98 - 14 = 84$

$\Rightarrow n - 1 = \frac{84}{7} = 12$

$\Rightarrow n = 13$

Hence, there are 13 numbers of two digit which are divisible by 7.

OR

If p^{th} term of an AP is q and the q^{th} term is p, prove that its n^{th} term is (p + q – n).

Ans : Let, a be first term and d be the common difference of the given AP. Then,

p^{th} term = q $\Rightarrow a + (p-1)d = q$ .... (i)

q^{th} term = p $\Rightarrow a + (q-1)d = p$ .... (ii)

Subtracting eq^n. (ii) from (i), we get,

$(p-q)d = -(p-q)$

$\Rightarrow d = -1$

Putting d = –1 in equation (i), we get

$a + (p-1)(-1) = q$

$\Rightarrow a = p + q - 1$

Thus, n^{th} term $= a + (n-1)d$

$= (p + q - 1) + (n-1)(-1)$

$= p + q - 1 - n + 1$

n^{th} term $= p + q - n$

Proved.

Section : C (Each 3 Marks)

Q.14 : The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1:3. Calculate the first and the thirteenth term of the AP.

Ans : Let a be the first term and d be the common difference of the given AP. Then,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = 42 \Rightarrow \frac{6}{2}[2a + (6-1)d]$$

$\Rightarrow 2a + 5d = 14$ .... (i)

It is given that,

$$\frac{a_{10}}{a_{30}} = \frac{1}{3}$$

$$\Rightarrow \frac{a + 9d}{a + 29d} = \frac{1}{3}$$

$\Rightarrow 3a + 27d = a + 29d$

$\Rightarrow 2a = 2d$

$\Rightarrow a = d$ .... (ii)

From (i) & (ii)

$\Rightarrow 2a + 5a = 14 \quad \Rightarrow 7a = 14 \quad \Rightarrow a = 2$

i.e. $a = d = 2$

$\therefore a_{13} = a + 12d = 2 + 12 \times 2 = 26$

Hence, first term = 2 and 13^{th} term = 26

OR

In a flower bed there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there in the flower bed?

Ans : The number of rose plants in first, second, third .... and last row are respectively.

23, 21, 19, ... 5 is an AP. with first term = 23, common difference = –2

i.e. $a = 23$, $d = -2$.

$\therefore a_n = a + (n-1)d$

$\Rightarrow 5 = 23 + (n-1)(-2)$

$\Rightarrow 2n = 20$

$\Rightarrow n = 10$

Hence, there are 10 rows of rose plants.

Q.15 : If p^{th}, q^{th} and r^{th} terms of an AP. are a, b and c respectively then prove that $a(q-r) + b(r-p) + c(p-q) = 0$.

Ans : Let A be the first term and

D be common difference of the given AP.

Then, $a = A + (p-1)D = p^{th}$ term

$b = A + (q-1)D = q^{th}$ term

$c = A + (r-1)D = r^{th}$ term

Now, we have,

$LHS = a(q-r) + b(r-p) + c(p-q)$

$= (A + (p-1)D)(q-r) + (A + (q-1)D)(r-p) + (A + (r-1)D)(p-q)$

$= A\{\cancel{q} - \cancel{r} + \cancel{r} - \cancel{p} + \cancel{p} - \cancel{q}\} + D\{(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)\}$

$= A \times 0 + D\{pq - q - pr + r + qr - r - pq + p + rp - p - rq + q\}$

$= A \times 0 + D \times 0$

$= 0 = RHS.$ Proved.

Section - D(Each 5 Marks)

Q.16 : The sums of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and their common difference are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : We have

S_1 = sum of n terms of an AP with first term 1 and common difference 1

$$\Rightarrow S_1 = \frac{n}{2}[2 \times 1 + (n-1) \times 1] = \frac{n}{2}(n+1)$$

S_2 = sum of n terms of on AP with first term 1 and fommon difference 2

$$\Rightarrow S_2 = \frac{n}{2}[2 \times 1 + (n-1)2] = n^2$$

S_3 = sum of n terms of an AP with first term 1 and common difference 3

$$\Rightarrow S_3 = \frac{n}{2}[2 \times 1 + (n-1)3] = \frac{n}{2}(3n-1)$$

$$\text{Now, } S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$$

$$= \frac{n}{2}[n + 1 + 3n - 1] = \frac{n}{2} \times 4n = 2n^2$$

$$= 2S_2 \qquad (\because S_2 = n^2)$$

Hence, $S_1 + S_3 = 2S_2$

OR

The ratio of the sums of m and n terms of an AP is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} terms is $(2m-1) : (2n-1)$.

Ans : Let a be the first term and d be common difference of the given AP, then the sums of m and n terms are given by

$$S_m = \frac{m}{2}[2a + (m-1)d] \text{ and}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{Then, } \frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$\Rightarrow n[2a + (m-1)d] = m[2a + (n-1)d]$

$\Rightarrow 2a\,(n-m) = d(mn - m - mn + n)$

$\Rightarrow 2a\,(n-m) = d(n-m)$

$\Rightarrow 2a = d$

$\therefore$ The ratio of m^{th} and n^{th} terms

$$\text{i.e. } \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a}$$

$$= \frac{a+2am-2a}{a+2an-2a}$$

$$= \frac{2am-a}{2an-a}$$

$$= \frac{\cancel{a}(2m-1)}{\cancel{a}(2n-1)}$$

$$= \frac{2m-1}{2n-1}$$

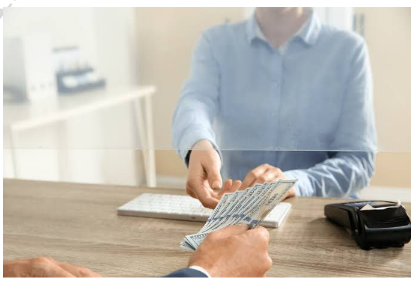
$$\therefore \frac{a_m}{a_n} = \frac{2m-1}{2n-1}$$

Proved.

Section : E

Q.17 : **Case study :**

Jaspal takes a loan from a bank for his car. Jaspal Singh repays his total loan of 118000 by paying every month starting with the first installment of 1000. If he increases the installment by 100 every month.



i) If the given problem is based on AP, then what is the first term and common difference? 1

Ans : Firt term = 1000

and common difference = 100

ii) In how many months the loan will be cleared? 2

Ans : Let the total months in which loan will be cleared = n

$$\text{So, } \frac{n}{2}[2\times1000+(n-1)100] = 118000$$

$$\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$$\Rightarrow \frac{n}{2}[20+n-1] = 1180$$

$$\Rightarrow n^2 + 19n - 2360 = 0$$

$$\Rightarrow n^2 + 59n - 40n - 2360 = 0$$

$$\Rightarrow (n+59)(n-40) = 0$$

$\Rightarrow n = -59$ (not possible), $n = 40$

Therefore, the no. of months will be 40 months.

OR

Find the amount paid by him in 30 installments.

Ans : The amount paid by him in 30 installments

$$= \frac{30}{2}[2\times1000+(30-1)100]$$

$$= 15\times100[20+29]$$

$$= 1500\times49$$

$$= ₹\ 73500$$

iii) Find the amount paid by him in 20^{th} installment. 1

Ans : The amount paid by him in 20^{th} instalments = $1000 + (20-1)\,100$

$= 1000 + 19 \times 100$

$= 1000 + 1900$

$= ₹\ 2900$

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