SHIKSHA CLASSES

Subject : Algebra
Class : X

Answer Paper

3. Arithmetic Progression

Total Marks : 20

Q.1 A) Choose the correct alternatives of the following questions. **2**

1) **In an A. P. the common difference denoted is by 'd'**

Ans: d) All the above.

2) **The fifth term of an A. P. is**

$$\frac{1}{2},\frac{1}{6},\frac{1}{18},\frac{1}{54}-------$$

Ans: b) $\frac{1}{162}$

B) Define sequence with example. **1**

Ans: Sequence : It is defined as, a set of numbers where the numbers are arranged in a definite order is called sequence. e.g.4, 8, 12, 16

Q. 2 A) : Attempt Any ONE of the following. **2**

1) **Write whether the following sequences is in A. P. ? If it is in A. P. find the common difference.**

i) 2, 4, 6, 8

Ans: Given Sequence is,

2, 4, 6, 8, -----

$\therefore\ t_1 = 2, t_2 = 4, t_3 = 6, t_4 = 8$

$\therefore\ t_2 - t_1 = 4 - 2 = 2$

$t_3 - t_2 = 6 - 4 = 2$

$t_4 - t_3 = 8 - 6 = 2$

$\therefore$ By defination of A. P. the difference bet^n two consecutive term is common i.e. 2

$\therefore$ The given sequence is A. P. and common difference is 2.

2) **Which term of the following A. P. is 560? 2, 11, 20, 29 -----------**

Ans. : Given A. P. is

2, 11, 20, 29 -------

$\therefore$ n^{th} term of this A. P. is 560

$t_n = a + (n-1)\ d$

$\therefore\ \boxed{560} = 2 + (n-1) \times \boxed{9}$

$\therefore\ 560 = 2 + 9n - 9$

$\therefore\ 560 = 2 - 9 + 9n$

$\therefore\ 560 = -7 + 9n$

$\therefore\ 560 + 7 = 9n$

$\therefore\ 567 = 9n$

$\therefore\ \frac{567}{9} = n$

$\therefore n = \boxed{63}$

$\therefore$ 63^{rd} term of given A. P. is $\boxed{560}$

Q. 2 B) : Attempt Any ONE of the following. **2**

1) **The first term 'a' and common difference 'd' are given. Find first four terms of A.P. a = –3 , d = 4.**

Ans. Given a = –3, d = 4

$t_1 = -3$

$t_2 = t_1 + d = -3 + 4 = 1$

$t_3 = t_2 + d = 1 + 4 = 5$

$t_4 = t_3 + d = 5 + 4 = 9$

$\therefore$ A.P. is –3, 1, 5, 9

2) Find t_n for following A.P.

3, 8, 13, 18, ...

Ans. Given A.P. is 3, 8, 13, 18, ...

Here a = 3, d = 8 – 3 = 5

$t_n = a + (n - 1) \times d$

$= 3 + (n - 1) \times 5$

$= 3 + 5n - 5$

$t_n = 5n - 2$

$\therefore t_n = 5n - 2$

Q.3 A) : Attempt Any ONE of the following. 3

1) Find the sum of the first 'n' odd natural numbers. Hence find 1 + 3 + 5 + — + 101.

Ans. : 1, 3, 5 ----- are the odd natural numbers

$\therefore\ t_1 = a = 1, d = 2;$

$\therefore S_n = \frac{n}{2}\left[2a + (n-1) \times d\right]$

$= \frac{n}{2}\left[2 \times 1 + (n-1) \times 2\right]$

$= \frac{n}{2}\left[2 + (n-1) \times 2\right]$

$= \frac{n}{2}\left[\cancel{2} + 2n - \cancel{2}\right]$

$= \frac{n}{\cancel{2}} \times \cancel{2}n$

$\therefore S_n = n \times n = n^2$ ________ (1)

Now,

$\therefore t_n = a + (n-1) \times d$

101 = 1 + (n-1) x 2

101 = 1 + 2n -2

101 = 1-2 + 2n

101 = -1 + 2n

101 + 1 = 2n

102 = 2n

$\frac{102}{2} = n$

$\therefore\ n = 51$

$\therefore$ From eq^n (1)

$Sn = n^2$

$\therefore S_{51} = (51)^2 = 2601$

$\therefore$ The sum of the first n odd natural numbers is n^2 ; $S_{51} = 2601$

2) Sum of first 55 terms in an A. P. is 3300, Find it's 28^{th} term.

Ans: $S_n = S_{55} = 3300$

$\therefore S_n = \frac{n}{2}\left[2a + (n-1) \times d\right]$

$\therefore = \frac{\boxed{55}}{2}\left[2a + \boxed{(55-1)} \times d\right]$

$= \frac{55}{2}\left[2a + 54d\right]$

$\therefore \boxed{3300} = \frac{55}{2} \times 2\left[a + 27d\right]$

$\therefore$ 3300 = 55(a + 27d)

$\therefore\ a + 27d = \boxed{60}$

$\therefore$ a + 27d = 60 ________ (1)

$\therefore$ We have to find out t_{28}

$t_n = a + (n - 1) \times d$

$\therefore t_{28} = a + (28-1) \times d$

$t_{28} = a + 27d$

From $eq^n (1)$

$\therefore a + 27d = 60$

$\therefore \ t_{28} = \boxed{60}$

$\therefore$ The 28th term is $\boxed{60}$.

B) Attempt Any ONE of the following. 3

1) The taxi fare is ₹ 14 for the first kilometere and ₹ 2 for each additional kilmetere. What will be the fare for 10 kilometers?

Ans: The increase in fare for each additional kilometere is ₹ 2

$\therefore d = 2$

The fare for the first kilometere is ₹ 14

$\therefore a = 14$

$\therefore$ We have to find the fare for 10 kilometers i.e. $t_{10} = ?$

$\therefore t_n = a + (n-1)d$

$\therefore t_{10} = 14 + (10-1) \times 2$

$= 14 + 9 \times 2$

$= 14 + 18$

$\therefore t_{10} = 32$

$\therefore$ The fare for 10 km will be ₹ 32.

2) Check whether 301 is in sequence.

5, 11, 17, 23, ... ?

Ans.: In the sequence 5, 11, 17, 23, ...

$t_1 = 5, t_2 = 11, t_3 = 17, t_4 = 23$

$t_2 - t_1 = 11 - 5 = 6$

$t_3 - t_2 = 17 - 11 = 6$

$\therefore$ This sequence is an A.P.

First term a = 5 and d = 6.

If 301 is n^{th} term then.

$t_n = a + (n-1) \times d = 301$

$\therefore \ 301 = 5 + (n-1) \times 6$

$= 5 + 6n - 6$

$6n = 301 + 1 = 302$

$\therefore \ n = \frac{302}{6}$. But it is not an integer

$\therefore$ 301 is not in the given sequence.

Q. 4 : Attempt Any ONE of the following. 4

1) Find four consecutive terms in an A. P. whose sum is 12 and sum of 3^{rd} and 4^{th} term is 14.

Ans. : Let the four consecutive tems in an A. P. be a-3d, a-d, a + d and a +3d

$\therefore$ By first condition.

$(a-3d) + (a-d) + (a+d) + (a+3d) = 12$

$a - \cancel{3d} + a - \cancel{d} + a + \cancel{d} + a + \cancel{3d} = 12$

$a + a + a + a = 12$

$\therefore \ 4a = 12$

$a = \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{4}}}$

$\therefore a = 3$

$\therefore$ By second condition

$(a+d) + (a+3d) = 14$

$a + d + a + 3d = 14$

$2a + 4d = 14$

$a + 2d = \frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{2}}}$

$a + 2d = 7$

$\therefore$ Put $a = 3$ in $a + 2d = 7$

$\therefore a + 2d = 7$

$3 + 2d = 7$

$2d = 7-3$

$2d = 4$

$d = 2$

$\therefore$ Substituting $a = 3$ and $d=2$ in the four tems.

$a - 3d = 3 - 3 \times 2 = 3 - 6 = -3$

$a - d = 3 - 2 = 1$

$a + d = 3 + 2 = 5$

$a + 3d = 3 + 3 \times 2 = 3 + 6 = 9$

The four consecutive terms are -3, 1, 5 and 9

2) The 10th term and 18th term of an A. P. are 25 and 41 respectively then find 38th term of that A. P. similarly if nth term is 99. Find the value of n.

Ans: In Given A. P.

$t_{10} = 25$ and $t_{18} = 41$

$t_n = a + (n-1) \times d$

$\therefore t_{10} = a + (10-1) \times d$

$\therefore 25 = a + 9d$ ______ (1)

Similarly $t_{18} = a + (18-1) \times d$

$41 = a + 17d$ _____ (2)

$\therefore$ From eq^n ______ (1)

$25 = a + 9d$

$25 - 9d = a$

Put $a = 25 - 9d$ in eq^n (2)

$\therefore a + 17d = 41$

$\therefore 25 - 9d + 17d = 41$

$25 + 8d = 41$

$\therefore 8d = 41 - 25$

$\therefore 8d = 16$

$\therefore$ Put $d = 2$ in eq^n (1)

$a + 9d = 25$

$a + 9 \times 2 = 25$

$a + 18 = 25$

$a = 25 - 18 = 7$

$\therefore a = 7$

$\therefore t_n = a + (n-1) \times d$

$\therefore t_{38} = 7 + (38 - 1) \times 2$

$= 7 + 37 \times 2$

$t_{38} = 7 + 74$

$\therefore t_{38} = 81$

$\therefore$ nth term is 99

$\therefore t_n = a + (n-1) \times d$

$99 = 7 + 2n - 2$

$99 = 5 + 2n$

$99 - 5 = 2n$

$\therefore 2n = 94$

$\therefore n = \frac{94}{2}$

$\therefore n = 47$

$\therefore$ 38th term is 81 and 99 is the 47th term

Q. 5 : Attempt Any ONE of the following. 3

1) How many three digit Natural numbers are divisible by four?

Ans: The smallest and the biggest three digit numbers divisible by four are 100 and 996 respectively

$\therefore$ The A. P. becomes,

100, 104, 108, ---- 996

$\therefore a = 100; d = 4, t_n = 996$

$\therefore t_n = a + (n-1) \times d$

$\therefore 996 - 100 = (n-1) \times 4$

$896 = 4n - 4$

$896 + 4 = 4n$

$900 = 4n$

$\frac{900}{4} = n$

$\therefore n = 225$

There are 225 three - digit natural numbers divisible by 4.

2) Find three consecutive terms in an A. P. whose sum is -3 and the product of their cubes is 512.

Ans. : Let the three consecutive terms be a - d, a and a + d

$\therefore$ by first condition,

(a - d) + a + (a + d) = -3

$a - \cancel{d} + a + a + \cancel{d} = -3$

a + a + a = -3

$a = \frac{-3}{3}$

$\therefore a = -1$

$\therefore$ By second condition.

$(a-d)^3 \times a^3 \times (a+d)^3 = 512$

Put a = -1 in above eq^n

$\therefore (-1-d)^3 \times (-1)^3 \times (-1+d)^3 = 512$

$\therefore [(-1)(-1-d)]^3 (-1+d)^3 = 512$

$\therefore (1+d)^3 (-1+d)^3 = 512$

$(1+d)^3 (-1+d)^3 = (8)^3$

Taking cube root on both sides

(1+d) (-1+d) = 8

$(d)^2 - (1)^2 = 8$

$\therefore d^2 - 1 = 8$

$d^2 = 8 + 1$

$d^2 = 9$

$\therefore d = \pm 3$

Taking a = -1 and d = 3

$\therefore (a-d) = -1-3 = -4;\ a = -1$

a + d = -1 + 3 = 2

$\therefore$ The terms are - 4, -1 and 2

Now, Taking a = -1 and d = -3

$(a-d) = -1-(-3) = -1+3 = 2$

a = -1

a + d = -1 -3 = - 4

$\therefore$ The three consecutive terms are

-4, -1 and 2 OR 2, -1 and -4

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