SHIKSHA CLASSES

Subject : Geometry **ANSWERS PAPER** **Total Marks** : 20
Class : X **2. Pythagoras theorem**

Q.1 : A) Choose the correct alternative from objectives given below. (2)

1) In ΔABC, M is the midpoint of side BC. If $AB^2 + AC^2 = 410cm^2$ and BC = 12cm, then what is the length of median AM?

Ans. : d) 13cm

2) What is the length of hypotenuse of a right angled triangle, if length of sides forming right angle are 9cm and 12cm?

Ans. : c) 15cm

B) Solve Any ONE of the following. (1)

1) In ΔLMN, ℓ=5, m=13, n=12. State whether ΔLMN is a right angled triangle or not.

Ans. : $\ell = 5, m = 13, n = 12$

$\ell^2 = 25, m^2 = 169, n^2 = 144$

$\therefore m^2 = \ell^2 + n^2$

$\therefore$ By converse of pythagorus theorem ΔLMN is a right angled triangle.

2) Find the side of a square whose diagonal is 10cm.

Ans. : Side of square $= \frac{1}{\sqrt{2}} \times$ diagonal

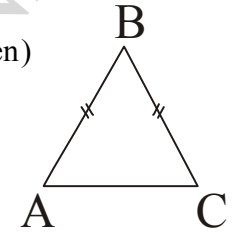
$= \frac{1}{\sqrt{2}} \times 10$

$= \frac{1}{\cancel{2}} \times \cancel{10} \times \sqrt{2} = 5\sqrt{2}$ cm

Q. 2 : A) Attempt any ONE of the following question. (2)

1) The hypotenuse of an isoceles right angled ΔABC is $8\sqrt{2}$ cm. Find BC.

Ans. : ΔABC is an isoceles right angled triangle, by pythagoras theorem.



$AC^2 = AB^2 + BC^2$

but $AB = BC$ ($\because$ given)

$AC^2 = BC^2 + BC^2$

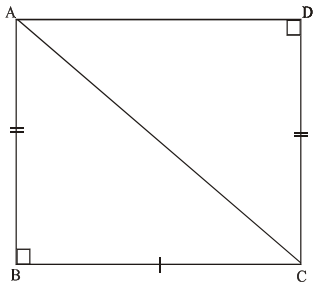
$AC^2 = 2BC^2$

$\left(8\sqrt{2}\right)^2 = 2BC^2$

$64 \times \cancel{2} = \cancel{2} BC^2$

$BC^2 = 64$, $BC = 8$cm

2) Find the diagonal of a rectangle whose length is 35cm and breadth is 12cm.



Ans. : Diagonal of rectangle divides in two right angled triangle

$\boxed{AC^2} = \boxed{AB^2} + \boxed{BC^2}$ [$\because$ Pythagorus Theorem]

$= 12^2 + 35^2$

$= 144 + 1225$

$= 1369$

$AC = \sqrt{1369} = \boxed{37}$

Diagonal of rectangle is $\boxed{37}$ cm.

Q. 2 : B) Attempt any ONE of the following question. (2)

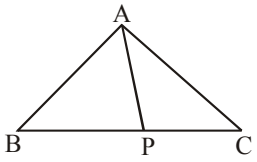
1) In ΔABC seg AP is a median. If BC = 18, $AB^2+AC^2=260$ Find AP.

Ans. : In ΔABC, seg AP is a median

$\therefore$ By Apollonius theorem

$$AB^2 + AC^2 = 2AP^2 + 2BP^2$$

but $BP = \frac{1}{2}BC = \frac{1}{2}\times 18 = 9$



$$\therefore 260 = 2AP^2 + 2BP^2$$
$$130 = AP^2 + 9^2$$
$$AP^2 = 130 - 81$$
$$AP^2 = 49$$
$$AP = \sqrt{49} = 7$$

Length of AP is 7 unit.

2) In $\Delta RST, \angle S = 90^0, \angle T = 30^0$, RT=12cm, then find RS and ST.

Ans. : ΔRST is 30^0-60^0-90^0 triangle

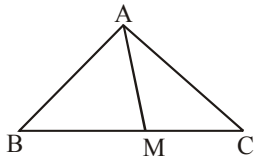
By 30^0-60^0-90^0 theorem

$$RS = \frac{1}{2}\times RT \qquad ST = \frac{\sqrt{3}}{2}\times RT$$
$$= \frac{1}{2}\times 12 \qquad = \frac{\sqrt{3}}{2}\times 12$$
$$= 6\,cm \qquad = 6\sqrt{3}\,cm$$

Q. 3: A) Attempt any ONE of the following questions. (3)

1) Seg AM is a median of ΔABC. If AB=22, AC = 34, BC = 24, Find AM.

Ans. : In ΔABC, AM is a median



$\therefore$ M is midpoint of side BC

$\therefore$ BM = CM

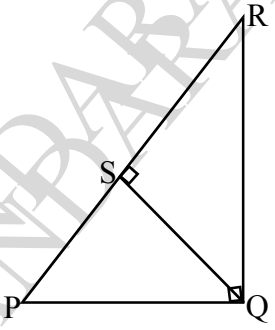
$$= \frac{1}{2}\times BC = \frac{1}{2}\times 24 = 12$$

$\therefore$ By Appolloius theorem

$$AB^2 + AC^2 = 2AM^2 + 2BM^2$$
$$22^2 + 34^2 = 2AM^2 + 2\times 12^2$$
$$484 + 1156 = 2AM^2 + 2 \times 144$$
$$242 + 578 = AM^2 + 144$$
$$820 = AM^2 + 144$$
$$AM^2 = 820 - 144 = 676$$
$$AM = \sqrt{676} = 26$$

2) State and prove theorem of geometric mean.

Ans. : In a right angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.



Proof : In right angled triangle PQR,

Seg QS $\perp$ hypotenuse PR

$\Delta QSR \sim \Delta PSQ$ ---[Similarity of right triangles]

$$\frac{QS}{PS} = \frac{SR}{SQ}$$

$$\frac{QS}{PS} = \frac{SR}{QS}$$

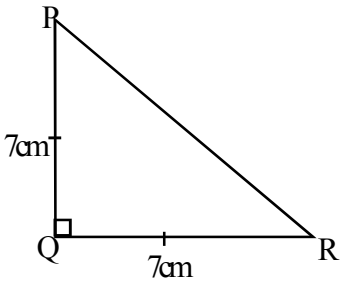
$$QS^2 = PS \times SR$$

$\therefore$ seg QS is the 'geometric mean' of seg PS and SR.

Q. 3 : B) Attempt any ONE of the following questions. (3)

1) Find the perimeter of an isosceles right angled triangle with each of the congruent side measuring 7cm.

Ans : ΔPQR is an isoceles right angle triangle



$\angle PQR = 90^0$

PQ = QR = 7cm

$\therefore\ PR^2 = PQ^2 + QR^2$ (Pythagorus th^m)

$= 7^2 + 7^2 = 49 + 49 = 98$

$PR = \sqrt{98} = \sqrt{2 \times 49} = 7\sqrt{2}$ cm

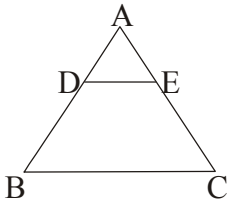
Perimeter of $\Delta PQR = PQ + QR + PR$

$= 7 + 7 + 7\sqrt{2}$

$= 14 + 7\sqrt{2}$

$= 7\left(2 + \sqrt{2}\right)$cm

2) In $\Delta ABC, DE \parallel BC$ If DB = 5.4 cm, AD = 1.8 cm EC = 7.2 cm. then find AE.



Ans : In $\Delta ABC, DE \parallel BC$

$\frac{AD}{DB} = \frac{AE}{EC}$

Basic proportionality theorem.

$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$

$\therefore AE \times 5.4 = 1.8 \times 7.2$

$\therefore AE = \frac{1.8 \times 7.2}{5.4}$

$AE = 2.4$cm .

Q. 4 : Solve Any ONE of the following. (4)

1) State and prove Pythagoras theorem.

Ans. : In a Right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

Given : In $\Delta ABC, \angle ABC = 90^0$

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw perpendicular seg BD on side AC A-D-C.

Proof : In right angled

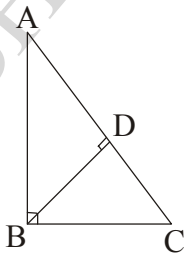
ΔABC, Seg BD $\perp$ hypotenuse AC

$(\because$ construction$)$

$\therefore \Delta ABC \sim \Delta ADB \sim \Delta BDC$ [Similarity of right angled traingles]

$\Delta ABC \sim \Delta ADB$

$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB}$ corresponding sides.



$\frac{AB}{AD} = \frac{AC}{AB}$

$AB^2 = AD \times AC$ __ (1)

Similarly, $\Delta ABC \sim \Delta BDC$

$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC}$ corresponding sides

$\frac{BC}{DC} = \frac{AC}{BC}$

$BC^2 = DC \times AC$ _____ (2)

Adding (1) and (2)

$AB^2 + BC^2 = AD \times AC + DC \times AC$

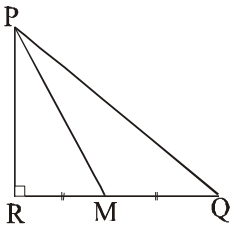
$= AC \times (AD + DC)$

$= AC \times AC$ _____ $[A - D - C]$

$\therefore AB^2 + BC^2 = AC^2$

$\therefore AC^2 = AB^2 + BC^2$

2) In the adjoining figure M is the midpoint of QR. $\angle PRQ = 90^0$ Prove that, $PQ^2 = 4PM^2 - 3PR^2$



Ans. : In ΔPRQ, M is mid-point of QR

$\therefore RM = MQ$

$\therefore RQ = 2RM = 2MQ$ _____ (1)

In ΔPRM, $\angle PRM = 90^0$,

by pythagorus theorem

$PM^2 = PR^2 + RM^2$

$RM^2 = PM^2 - PR^2$ _____ (2)

In ΔPRQ, $\angle PRQ = 90^0$,

by pythagorus theorem

$PQ^2 = PR^2 + RQ^2$

$= PR^2 + (2RM)^2$ [$\because$ from (1)]

$= PR^2 + 4RM^2$

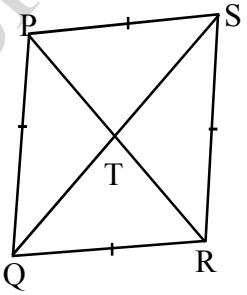
$= PR^2 + 4(PM^2 - PR^2)$ [$\because$ from (2)]

$= PR^2 + 4PM^2 - 4PR^2$

$\therefore \boxed{PQ^2 = 4PM^2 - 3PR^2}$

Q. 5 : Solve Any ONE of the following. (3)

1) Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.



Ans. : Given : □PQRS is a rhombus. Diagonal PR and SQ intersect each other at point T.

To prove :

$PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

Proof. : Diagonals of a rhombus bisect each other, By appollonius theorem,

$PQ^2 + PS^2 = 2PT^2 + 2QT^2$ _____ I

$QR^2 + SR^2 = 2RT^2 + 2QT^2$ _____ II

$\therefore$ Adding (I) and (II)

$PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + RT^2) + 4QT^2$

$= 2(PT^2 + PT^2) + 4QT^2$ _____ $(RT = PT)$

$= 4PT^2 + 4QT^2$

$= (2PT)^2 + (2QT)^2$

$= PR^2 + QS^2$

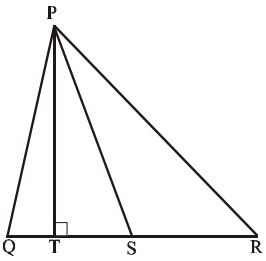
2) In the adjoining figure seg PS is the median of ΔPQR and $PT \perp QR$ prove that.

i) $PR^2 = PS^2 + QR \times ST + \left[\frac{QR}{2}\right]^2$

Ans. : In ΔPQR, seg PS is median

$\therefore$ S is mid-point of QR

$\therefore QS = SR = \frac{1}{2}QR$ _____ (1)



In ΔPTS, $\angle PTS = 90^0$

$\therefore PS^2 = PT^2 + TS^2$ [Pythagorus theorem]

$PT^2 = PS^2 - TS^2$ _____ (2)

In ΔPTR, $\angle PTR = 90^0$

$\therefore PR^2 = PT^2 + TR^2$ (Pythagorus theorem)

$= PT^2 + [TS + SR]^2$ [T – S – R]

$= PT^2 + TS^2 + 2ST \times SR + SR^2$

$= PT^2 + TS^2 + 2 \times ST \times \frac{QR}{2} + \left[\frac{QR}{2}\right]^2$ [from (1)]

$= PT^2 + TS^2 + ST \times QR + \left(\frac{QR}{2}\right)^2$

$= PS^2 - TS^2 + TS^2 + ST \times QR + \left(\frac{QR}{2}\right)^2$ (from (2))

$\boxed{PR^2 = PS^2 + ST \times QR + \left[\frac{QR}{2}\right]^2}$