

SHIKSHA CLASSES

Subject: Physics

BOARD ANSWER PAPER

Total Marks: 20

Class : XII

Topic: 8. Electrostatics

Section (A)

- Q.1 : A) Select and write the most appropriate answer from given alternatives in each sub-question. [5]
 - 1) There are two charges $+1\,\mu C$ and $+5\,\mu C$ the ratio of the forces acting on them will be..

Ans.: b) 1:1

$$q_1 = 1\mu C$$
 $q_2 = 5\mu C$

$$F_1 = \frac{1}{4\pi \in_0} \frac{\left(1 \times 10^{-6}\right) \left(5 \times 10^{-6}\right)}{r^2} ---(i)$$

$$F_2 = \frac{1}{4\pi \in {}_0} \frac{\left(5 \times 10^{-6}\right) \left(1 \times 10^{-6}\right)}{r^2} ---(ii)$$

divide eqn (i) by eqn (ii)

$$\frac{F_1}{F_2} = \frac{1}{1}$$
 but direction of F_1 and F_2 is different.

2) A capacitor of capacity C has a charge Q and stored energy is W. If the charge is increased to 2Q, the stored energy will be...

Ans.: c) 4W

Energy stored W =
$$\frac{Q^2}{2C}$$

∴ W \propto Q² (if C is constant)

$$W \propto Q^2$$
 $---(i)$

$$W' \propto (2Q)^2 ---(ii)$$

$$\frac{W}{W'} = \frac{Q^2}{(2Q)^2} = \frac{Q^2}{4Q^2} \quad \boxed{W' = 4W}$$

3) The energy stored in a condenser is in the form of

Ans.: b) Potential energy

4) A metal plate of area 0.5 m^2 is given a charge of $50\mu C$. The mechanical force acting on the metal plate is

Ans.: c) 565 N/m^2

5) In n condensers each of capacity C are connected in series, effective capacity of combination is

Ans.: a) $\frac{C}{n}$

Q.1 : B) Very short answer type question.

[2]

1) A wire is bent in a circle of radius 20 cm. it is given a charge of 200 μ C. Which spread on it uniformly. What is electric potential at the centre?

Ans.: Given:

$$q = 200 \mu C = 200 \times 10^{-6} C$$

$$R = 20 \text{ cm} = 2 \times 10^{-2} \text{ m V} = ?$$

$$As V = \frac{1}{4\pi \in_0} \frac{q}{r}$$

$$=\frac{9\times10^{9}\times\cancel{200}\times10^{-6}}{\cancel{2}\times10^{-2}}$$

$$=9\times10^{9}\times10^{2}\times10^{-6}\times10^{2}$$

$$= 9 \times 10^{13-6} = 9 \times 10^7 \text{ Volt}$$

2) Explain the concept of potential.

Ans.: Concept of Potential: The potential energy is the work done against the electrostatic forces to achieve a certain configuration of charges in a given system. Since every system tries

Consider a positive charge Q fixed at some point in space. For bringing any

other positive charge close to it, work is necessary. This work is equal to the change in the potential energy of their system.

Thus, work done against a electrostatic force - Increase in potential energy of the system.

$$\therefore \vec{F}d\vec{r} = dU$$
,

Where dU is the increase in potential energy when charges is displaced through $d\vec{r}$ and \vec{F} is the force exerted on the charge.

Section (B)

Q.2 : Attempt any THREE. [6]

1] Define: i)Electric flux ii) Capacitance

Ans:

i) Electric flux: The number of lines of force passing through per unit area is called electric flux, S.I. unit - Vm. a given area around a point in an electric field is called electric flux.

- ii) Capacitance: The capacitance of a capacitor is the ratio of magnitude of charge on either of conductor to the magnitude of potential difference between two conductors.
- 2] Define capacity of a condenser and state its S.I. unit.

Ans: The capacitance of a capacitor is the ratio of magnitude of charge on either of conductor to the magnitude of potential difference between two conductors.

$$C = \frac{Q}{V}$$
 where $C = Capacity, Q =$

Charge,

V =potential difference.

S.I. unit is farad.

A capacitor has a capacitance of one farad, if the potential difference across it rises by 1 volt, when 1 coulomb of charge is given to it.

3] Calculate the surface density of charge on a metal sphere of diameter 10 cm situated in air, if electric intensity at a point of 15 cm is 2 x 10⁴ N/C.

Ans: Given:

$$D = 10 \text{ cm}, R = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$k = 1, r = 15 \times 10^{-2} \text{ m}, E = 2 \times 10^{4} \text{ N/c}$$
 To find $\sigma = ?$

$$\begin{split} E &= \frac{\sigma R^2}{\varepsilon_0 r^2} \\ \sigma &= \frac{k \varepsilon_0 r^2 E}{R^2} \\ &= \frac{1 \times 8.85 \times 10^{-12} \times (15 \times 10^{-2})^2 \times 2 \times 10^4}{(5 \times 10^{-2})^2} \\ &= \frac{8.85 \times 10^{-12} \times 225 \times 10^{-4} \times 2 \times 10^4}{25 \times 10^{-4}} \\ &= 8.8 \times 9 \times 2 \times 10^{-8} \end{split}$$

4] Explain the concept of a condenser.

 $\sigma = 1.593 \times 10^{-6} \,\mathrm{C/m^2}$

Ans: P_1 P_2 P_1 P_2 P_3 P_4 P_5 P_5 P_6 P_6 P_7 P_8 P_8

Consider a metal plate P_1 having area A. Let positive charge Q be the charge on insulated metal plate P_1 , when it is fully charged let its potential is V.

$$\therefore C_1 = \frac{Q}{V} \qquad \dots (1)$$

Now another identical metal plate P_2 is kept near and parallel to plate P_1 . The inner surface metal P_2 acquires charge -Q and outer surface of metal plate P_2 acquires + Q charge by methodof induction. The

induced negative charge lowers the potential of P_1 , while the positive charge raises its potential. However negative charge is nearer to plate P_1 than positive charge. Therefore there is net decrease in potential of plate P_1 .

If outer surface of P is connected to the connected to the earth, the unbound positive charge + Q escapes to the earth.

Let $-V_1$ be the electric potential due to charge -Q on plate P_2 .

Hence, the net electric potential of system = $V + (-V_1) = V - V_1$

 \therefore The capacity in this case =

$$C_2 = \frac{Q}{V - V_1}$$
 (2)

Form equation (1) and (2), $C_2 > C_1$. Thus, if an earth connected conductor is brought near a charged conductor, its capacity increases. This is the principle of condenser.

Section (C)

Q. 3. Attempt any one

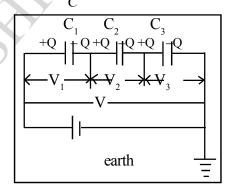
[3]

1. Derive the formula for effective capacity of three condensers connected in series.

Ans: Consider three condenser of capacities C_1 , C_2 , and C_3 are connected in series as shown in the figure.

Let V be the P. D. applied across the combination. Let the P.D. across C_1 , C_2 and C_3 is V_1 , V_2 , and V_3 respectively. Let Q be the total charge supplied by

$$V = V_1 + V_2 + V_3$$
(1)
But $V = \frac{Q}{C}$.



$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

Putting the values of V_1 , V_2 and V_3 in equation (1), we get:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V = Q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right] \quad \dots (2)$$

Let the effective capacity be $= C_s$

$$\therefore \quad \text{P.D. across } C_3 \text{ i.e. } V = \frac{Q}{C_s} \quad \dots (3)$$

From (2) and (3),

$$\frac{Q}{C_s} = Q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

$$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

i.e. reciprocal of effective capacity is the sum of reciprocals of individual capacities.

2] A condenser of capacity of 100 mF is charged up to a potential of 500 V. If the area of each plate of the condenser is 20 cm² and the distance between the plates is 1 mm. Find the energy density of the space between the plates.

Ans: Given: $C = 100mF = 100 \times 10^{-3} F = 10^{-1} F$, V = 500 Volt,

A = $20 \text{cm}^2 = 20 \times 10^{-4} \text{m}^2$, d = $1 \text{mm } 10^{-3} \text{ m}$ **To Find :** Energy density = ?

i) Total Energy
$$E = \frac{1}{2}CV^2$$

 $E = \frac{1}{2} \times 10^{-1} \times (500)^2$
 $\therefore E = \frac{25}{2} \times 10^{-1} \times 10^4$
 $\therefore E = 12.5 \times 10^3 \text{ J}$

ii) Energy density=

Total Energy

Volume of the space between the plates

$$= \frac{E}{A.d} = \frac{12.5 \times 10^3}{20 \times 10^{-4} \times 10^{-3}} = \frac{12.5}{2} \times 10^6 \times 10^3$$
$$= 6.25 \times 10^9 \text{ J/m}^3$$

Section (D)

Q. 4. Attempt any one

[4]

1) a. Derive the formula for energy stored in a charged condenser.

Ans: Consider a capacitor of capacity C.

Let the initial charge on the condenser be zero, so the potential of condenser will be zero.

In the process of charging, let the charge and potential be q and v in any intermediate stage.

$$\therefore C = \frac{q}{v}$$

$$\therefore \upsilon = \frac{q}{C} \qquad \dots \tag{1}$$

Suppose a small charge + dq is added to the condenser, the existing charge q will repel this charge + dq and hence work will be done in supplying the charge dq against the repulsive force.

Work done is given by : $d\omega = v \cdot dq$

Putting the value of v from equation (1)

$$\therefore d\omega = \frac{q}{C}dq$$

Let Q and V is the charge and potential when the condenser is fully charged. So to charge the condenser from 0 to Q the total work done will be obtained by integration.

$$\therefore \omega = \int_{0}^{Q} d\omega$$

$$\therefore \omega = \int_{0}^{Q} \frac{q}{C} dq$$

$$\therefore \omega = \frac{1}{C} \int_{0}^{Q} q \, dq$$

$$\therefore \omega = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q$$

$$\therefore \omega = \frac{1}{2C} \left[Q^2 - 0^2 \right]$$

$$\therefore \omega = \frac{1}{2C} \cdot Q^2 = \frac{Q^2}{2C}$$

This work done is stored in the condenser in form of electric potential energy in the space between the plates of condenser.

$$\therefore \quad \text{Energy of condenser} = \frac{Q^2}{2C} \dots (2)$$

Different Forms:

We know that
$$C = \frac{Q}{V}$$
 $\therefore Q = CV$

Putting the value of Q in equation (2),

Energy =
$$\frac{(CV)^2}{2C} = \frac{C^2V^2}{2C} = \frac{1}{2}CV^2$$

$$Energy = \frac{1}{2}CV^2$$

Also
$$C = \frac{Q}{V}$$

Putting this value in equation (3),

$$Energy = \frac{1}{2} \cdot \frac{Q}{V} \cdot V^2 = \frac{1}{2} Q \cdot V .$$

$$\therefore Energy of condenser = \frac{Q^2}{2C}$$

$$= \frac{1}{2}CV^2 = \frac{1}{2}QV$$

b. Three condensers with capacitance $10\mu F$, $20\mu F$ and

 $30\mu F$ are connected in series and a potential difference of 220 volt is applied across the combination. Find the resultant capacity, charge on each condenser and P.D. across each condenser.

Ans: Given:
$$C_1 = 10 \mu F$$
, $C_2 = 20 \mu F$, $C_3 = 30 \mu F$

To Find:
$$C_s = ?$$
, $Q_1 = ?$, $Q_2 = ?$, $Q_3 = ?$, $V_1 = ?$, $V_2 = ?$, $V_3 = ?$

Soluⁿ:
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore \frac{1}{C_s} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$$

$$\therefore \quad \frac{1}{C_s} = \frac{6+3+2}{60} \quad \therefore \quad \frac{1}{C_s} = \frac{11}{60}$$

$$\therefore C_s = \frac{60}{11} \mu F$$

$$Q_s = C_s \cdot V = \frac{60}{11} \times 10^{-6} \times 220$$

:.
$$Q_s = 1200 \times 10^{-6} C$$

Charge $Q_s = Q_1 = Q_2 = Q_3 = 1200 \times 10^{-6} C$

$$\therefore C_1 = \frac{Q_1}{V_1}$$

$$\therefore V_1 = \frac{Q_1}{C_1}$$

$$V_1 = \frac{1200 \times 10^{-6}}{10 \times 10^{-6}} = 120 \text{ V}$$

$$\therefore V_2 = \frac{Q_2}{C_2}$$

$$V_2 = \frac{1200 \times 10^{-6}}{20 \times 10^{-6}} = 60 \text{ V}$$

$$\therefore V_3 = \frac{Q_3}{C_2}$$

$$V_3 = \frac{1200 \times 10^{-6}}{30 \times 10^{-6}} = 40 \text{ V}$$

OR

2) a. Derive the formula for effective capacity of three condensers connected in parallel.

Ans: Consider the three condensers of capacities

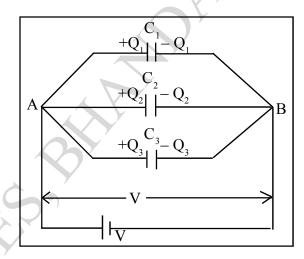
 C_1 , C_2 and C_3 connected in parallel as shown in the figure.

Let the total charge supplied by the cell be Q. This charge will be distributed among the three capacitors according to their capacities.

Let the charge on capacitors of capacities C_1, C_2 and C_3 be Q_1 , Q_2 and Q_3 respectively.

... The condensers are connected in parallel, hence the P.D. across each of them will be same and equal to total voltage V.

$$Q = Q_1 + Q_2 + Q_3$$
 (1)



$$C_1 = \frac{Q_1}{V}$$

$$Q_1 = C_1 V$$
, $Q_2 = C_2 V$, $Q_3 = C_3 V$

Putting the values of Q_1 , Q_2 and Q_3 in equation (1)

$$Q = C_1 V + C_2 V + C_3 V$$

$$Q = V(C_1 + C_2 + C_3) \qquad (2)$$

Let the effective capacity of combination is C_p

$$C_p = \frac{Q}{V}$$

$$Q = C_p V$$

Putting the value of Q in equation (2), we get

$$C_p V = V(C_1 + C_2 + C_3)$$

$$\therefore C_n = C_1 + C_2 + C_3$$

When the capacitors are connected in parallel their effective capacity is equal to the sum of individual capacitances.

b. A positively charged sphere of radius 10cm is surrounded by a medium of dielectric constant 5. If the magnitude of the electric field intensity at a point outside the sphere at a distance r from the centre of sphere is 6.4×10^{-4} V/m and charge on the sphere is 6.4×10^{-12} C, find r.

Ans: Given: $R = 10 \text{ cm} = 10 \times 10^{-2} \text{ cm}$

$$k = 5$$

$$q = 6.4 \times 10^{-12} \text{ C}$$

$$E = 6.4 \times 10^{-4} C$$

To Find: r = ?

Solution:

Electric Intensity $E = \frac{1}{4\pi \in_0 k} \cdot \frac{q}{r^2}$

$$\therefore r^2 = \frac{1}{4\pi \in_0 k} \cdot \frac{q}{E}$$

$$\therefore r^2 = \frac{9 \times 10^9}{5} \times \frac{\% \times 10^{-12}}{\% \times 10^{-4}}$$

$$\therefore r^2 = \frac{9}{5} \times 10^{13} \times 10^{-12}$$

$$r^2 = 1.8 \times 10^1$$

$$r^2 = 180$$

$$r = \sqrt{180}$$

$$\therefore r = 13.81m$$

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