SHIKSHA CLASSES

Subject : Physics **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 6. Superposition of Waves**

Section (A)

Q. 1 : A) Select and write the most appropriate answer from given alternatives in each sub-question. [5]

1) Two tuning fork of frequencies n_1 and n_2 produces n beats per second, if n_2 and n are known, n_1 may be given by.

Ans. : b) $n_2 \pm n$

Beat frequency = no of beat second

$$n = n_2 \sim n_1$$

$$\boxed{\therefore n_1 = n_2 \pm n}$$

2) Two waves having the intensities in the ratio of 9:1 produce interference. The ratio of maximum and minimum intensities is equal to.

Ans. : b) 4:1

$$\frac{I_{max}}{I_{min}} = \frac{\left(\frac{\sqrt{I_1}}{\sqrt{I_2}}+1\right)^2}{\left(\frac{\sqrt{I_1}}{\sqrt{I_2}}-1\right)^2} = \frac{\left(\sqrt{\frac{9}{1}}+1\right)^2}{\left(\sqrt{\frac{9}{1}}-1\right)^2} = \frac{4}{1}$$

3) A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillation of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of the sound in the air is 340 m/s.

Ans. : d) 6

$\Rightarrow$ for closed pipe

$$F = \frac{(2n+1)V}{4\ell}, \frac{(2n-1)V}{4\ell} < 1250$$

$$P = 85 \text{ cm}$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$\boxed{V = 340 \text{ m/s}}$ $(2n+1) < 12.5$

$$2n = 12.5 - 1 = 11.5$$

$$n < 5.25$$

So, $n = 0, 1, 3, 4, \ldots 5$

So we have 6 possibilities.

4) An empty vessels is partially filled with water, the frequency of vibration of air column in the vessel.

Ans. : c) Increases

Closed pipe, $n = \frac{V}{4\ell}(2N-1)$

$$n \times \frac{1}{\ell}$$

length of column decreases, frequency increases.

5) When temperature increases, the frequency of tuning fork..

Ans. : b) Decreases

Q1 : B) Very short answers type questions. [2]

1) What are the harmonics and overtones?

Ans. : **Harmonics :** Harmonics are the simply integral multiple of the fundamental frequency, All Harmonics may be present in a given sound.

Overtones : The higher frequencies of vibrating above the fundamental frequency are called overtones.

All Overtone are actually present in the given sound.

2) What are the beats? State its types.

Ans. : The alternate waxing and waning of sound at definite intervel of time due to superposition of two waves of nearly equal frequencies is called beats.

There are two types :

i) Waxing ii) Waning

Section (B)

Q.2 : Attempt any THREE. [6]

1) Define free and forced vibrations. State each of one example.

Ans. : **Force Vibration :** When a body, capable of osciliating is displaced from its stable equilibrium position and released it makes oscillation which are called free vibrations or natural frequency

Ex. Oscillations of a spring

Forced Vibrations : The vibrations of a body under action of external periodic force in which body vibrates with frequency equal to frequency of periodic force other than its natural frequency are called forced vibrations.

Ex. Vibrations of pendulum of clock.

2) Find the distance between two successive nodes in stationary wave on a string vibrating with frequency 75 Hz. The velocity of progressive wave that resulted in the stationary wave is 150 m/s.

Ans. : **Given :** Speed of wave = V = 150 m/s

frequency = 75 Hz

we have $V = n\lambda$

$$\therefore \lambda = \frac{V}{n} = \frac{150}{75} = 2\text{m}$$

We know the distance between successive nodes $= \frac{\lambda}{2} = \frac{2}{2} = 1\text{ m}$.

3) Two organ pipes of the same length, open at both ends produces sound of different frequencies, if their radii are different. Why?

Ans. : Suppose two organ pipes open at both ends and of same length.

Let d_1 and d_2 are their inner diameters which are not same the effective length of air column is,

$$\ell_1 = L + 0.3\,d_1 \text{ and } \ell_2 = L + 0.3\,d_2$$

Where 0.3 d_1 and 0.3 d_2 are end corrections

$$\text{Since, } n_1 = \frac{V}{2\ell_1} = \frac{V}{2(L + 0.3\,d_1)} \quad \text{---(i)}$$

$$n_2 = \frac{V}{2\ell_2} = \frac{V}{2(L + 0.3\,d_2)} \quad \text{---(ii)}$$

where V - velocity of sound in Air equation (i) & (ii), it is clear that the value of fundamental frequencies are different for diameters.

4) Two tuning fork of frequencies 340 Hz and 360 Hz produces sound waves of wavelength differing by 6 cm in medium. Find the velocity of sound in the medium?

Ans. : $n_1 = 340\text{ Hz}$ and $n_2 = 360\text{ Hz}$

wavelength difference

$= 6\text{ cm} = 6 \times 10^{-2}\text{ m}$

we have

$$n_1 < n_2$$

$$\lambda_1 > \lambda_2$$

$$\therefore \lambda_1 - \lambda_2 = 6 \times 10^{-2}$$

$$\lambda_1 - \lambda_2 = V\left[\frac{1}{n_1} - \frac{1}{n_2}\right]$$

$$6 \times 10^{-2} = V\left[\frac{1}{340} - \frac{1}{360}\right]$$

$$6 \times 10^{-2} = V\left[\frac{360 - 340}{340 \times 360}\right]$$

$$= V\left[\frac{20}{340\times 360}\right]$$

$$6\times 10^{-2} = \frac{20V}{340\times 360}$$

$$V = \frac{6\times 10^{-2}\times 340\times 360}{20}$$

$$V = \frac{6\times 10^{-2}\times 122400}{20}$$

$$V = \frac{6\times 10^{-2}\times 1224\times 10^{2}}{20}$$

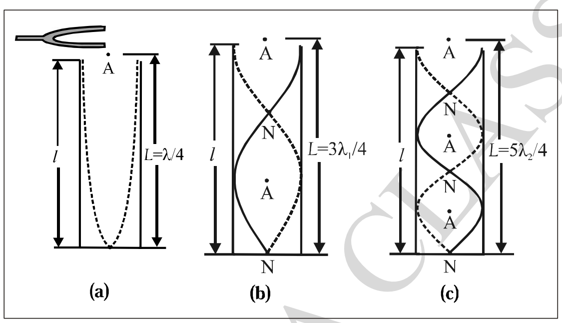
$\boxed{V = 367.2\ m/s}$.

Section (C)

Q.3 : Attempt any one of following. **[3]**

1) Prove that only odd harmonics are present in the vibration of the air column in a pipe closed at one end.

Ans. :



If longitudinal waves are sent along the air column in a pipe closed at one end, by holding a vibrating tunning fork near th open end, they get refleted at the closed end.

The air at the closed end is not free to vibrate and hence a node is formed at the closed end.

Air at the open end is free to vibrate and so an antinode is formed at the open end. The different ways in which the air column can vibrate are called modes of vibration of the air column.

– The simplest mode of vibration is called the first or fundamental mode of vibration.

– The distance between a node and consecutive antinode is $\lambda/4$ where λ is the wavelength of sound.

Here, $L = \frac{\lambda}{4} \quad \therefore \lambda = 4L$

But $V = n\lambda = n\times 4L$

$\therefore\ n = \frac{V}{4L}$ where n is the frequency of vibration at resonance.

In second mode of vibration, two antinodes and two nodes are formed.

$\therefore$ Length of air column

$\therefore L = \frac{3\lambda_1}{4}$ $\boxed{\therefore \lambda_1 = \frac{4L}{3_1}}$

λ_1 - is the wavelength of wave in second mode of vibration of air column.

let, n_1 be the frequency of wave in second mode of vibration of air column.

$\therefore V = n_1\lambda_1 \qquad \therefore n_1 = \frac{V}{\lambda_1} = \frac{3V}{4L}$

$\therefore n_1 = 3n \qquad \therefore \left(\because n = \frac{V}{4L}\right)$.

This frequency is called third harmonic or first overtone.

In third mode of vibration, of air column three antinode and three node are formed.

$\therefore$ Length of air column $= L = \frac{5\lambda_2}{4}$

$$\therefore \lambda_2 = \frac{4L}{5}$$

But $V = n_2\lambda_2$

Where, n_2 and λ_2 are the frequency and wavelength of the wave in third mode of vibration of air column,

$$\therefore \quad V = n_2 \frac{4L}{5} \qquad \therefore n_2 = \frac{5V}{4L} = 5\left(\frac{V}{4L}\right)$$

$$\therefore \boxed{n_2 = 5n}$$

This frequency is called the frequency of fifth harmonic or second overtone.

Thus, only odd harmonics are present as overtones in the case of an air column vibrating in a pipe closed at one end.

2) The length of sonometer wire between two fixed ends is 99 cm. where should be the two bridges are placed so as to divide the wire into three segments, whose fundamental frequencies are in the ratio 1:2:3?

Ans. : **Given :** $\ell = 99\text{cm}$

$$n_1 : n_2 : n_3 = 1 : 2 : 3$$

$$\frac{n_1}{n_2} = \frac{1}{2} \; \frac{n_2}{n_3} = \frac{2}{3}$$

$$\ell_1 = ?, \ell_2 = ?, \ell_3 = ?$$

Let ℓ_1, ℓ_2 and ℓ_3 be the length of three segments

$$\ell_1 + \ell_2 + \ell_3 = 99 \qquad \text{---(i)}$$

When T and m are same, then

$$n_1\ell_1 = n_2\ell_2 = n_3\ell_3$$

$$\frac{n_1}{n_2} = \frac{\ell_1}{\ell_2}$$

$$\frac{1}{2} = \frac{\ell_2}{\ell_1}$$

$$\boxed{\therefore \ell_2 = \frac{\ell_1}{2}} \text{ ---(ii)}$$

Similarly

$$\frac{n_2}{n_3} = \frac{\ell_3}{\ell_2}$$

$$\therefore \frac{2}{3} = \frac{\ell_3}{\ell_2}$$

$$\therefore \ell_3 = \frac{2}{3}\ell_2 = \frac{2}{3} \times \frac{\ell_1}{2}$$

$$\ell_3 = \frac{\ell_1}{3} \qquad \text{---(iii)}$$

Putting value of ℓ_2 and ℓ_3 in equation (i)

$$\therefore \; \ell_1 + \frac{\ell_1}{2} + \frac{\ell_1}{3} = 99$$

$$\frac{11}{6}\ell_1 = 99$$

$$\therefore \ell_1 = 99 \times \frac{6}{11} = 9 \times 6$$

$$\boxed{\ell_1 = 54 \text{ cm}}$$

$$\therefore \ell_2 = \frac{\ell_1}{2} = \frac{54}{2}$$

$\ell_2 = 27$ cm and

$$\ell_3 = \frac{\ell_1}{3} = \frac{54}{3}$$

$$\boxed{\ell_3 = 18 \text{ cm}}$$

Section (D)

Q.4 : Attempt any one. **[4]**

a) Show that even as well as odd harmonics are present as overtones in modes of vibration of string.

Ans. : a) Consider a string of length *l* stretched between two rigid supports. The linear density (mass per unit length of string) is m and the tension T acts on the string due to stretching. If it is made to vibrate by plucking or by using a vibrator like a tuning fork, a transverse wave can be produced along the string.

When the wave teaches to the fixed ends of the string, it gets reflected with change of phase by π radians. The reflected waves interfere with the incident wave and stationary wavs are formed along the string. The string vibrates with different modes of vibrations.

If a string is stetched between two rigid supports and is plucked at its centre, the string vibrates as shown in Fig. (a). It consists of an antinode formed at the centre and nodes at the two ends with one loop formed along its length. If λ is the wavelength and l is the length of the string, we get

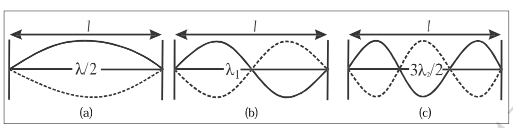
$$\text{Length of loop} = \frac{\lambda}{2} = l$$

$$\therefore \lambda = 2l$$

The frequency of vibration of the string,

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \left(\because v = \sqrt{\frac{T}{m}}\right)$$

This is the lowest frequency with which the string can vibrate. It is the fundamental frequency of vibrations or the first harmonic.



Different modes of vibrations of a stretched string

If the centre of the string is prevented from vibrating by touching it with a light object and string is plucked at a point midway between one of the segments, the string vibrates as shows in fig. (b).

Two loops are formed in this mode of vibrations. There is a node at the centre of the string and at its both ends. If λ_1 is wavelength of vibrations, the length of one loop

$$= \frac{\lambda_1}{2} = \frac{\ell}{2} \quad \therefore \lambda_1 = l$$

Thus, the frequency of vibrations is given as

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}}$$

$$n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$$

Comparing with fundamental frequency we get that $n_1 = 2n$.

Thus the frequency of the first overtone or second harmonic is equal to twice the fundamental frequency.

The string is made to vibrate in such a way that three loops are formed along the string as shown in fig. (c). If λ_2 is the wavelength here, the length of one loop is $\frac{\lambda_2}{2} = \frac{l}{3}$

$$\therefore \lambda_2 = \frac{2l}{3}$$

Therefore the frequency of vibrations is

$$n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}}$$

$$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

Comparing with fundamental frequency, we get that $n_2 = 3n$.

b) The speed of transverse wave along a uniform metal wire, when it is under the tension of 1000 g-wt is 68 m/s. if the density of metal is 7900kg/m³,find the area of cross section of the wire.

$$\Rightarrow T = 1000\text{ g-wt} = 1000 \times 10^{-3}\text{ kg wt}$$

$$= 1\text{ kg-wt} = 1 \times 9.8 = 9.8\text{N}$$

$$V = 68\text{m/s} \qquad \rho = 7900\text{ kg/m}^3 \quad A = ?$$

Formula - $V = \sqrt{\frac{T}{m}}$ ---(i)

mass of wire $(M) = V\rho = A\ell\rho$

mass per unit length $= m = \frac{M}{\ell}$

$$= \frac{A\ell\rho}{\ell} = A\rho$$

eq^n (i) becomes

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{A\rho}}$$

$$V^2 = \frac{T}{A\rho}$$

$$A = \frac{T}{V^2\rho} = \frac{9.8}{(68)^2 \times 7900}$$

$$\boxed{A = 2.683 \times 10^{-7} m^2}.$$

OR

a) Derive analytical method to determine beat frequency.

Ans. : a) **Analytical method to determine beat :**

Consider two sound waves, having same amplitude and slightly different frequencies n_1 and n_2. Let as same that they arrive in phase at same point x of the medium. The displacement due to each wave at any instant of time at that point is given as

$$y_1 = a \sin\left\{2\pi\left(n_1 t - \frac{x}{\lambda_1}\right)\right\}$$

$$y_2 = a \sin\left\{2\pi\left(n_2 t - \frac{x}{\lambda_2}\right)\right\}$$

Let us assume for simplicity that the listener is at $x = 0$.

$\therefore y_1 = a\sin(2\pi n_1 t)$ and

$y_2 = a\sin(2\pi n_2 t)$

According to the principle of superposition of waves,

$$y = y_1 + y_2$$

$$\therefore y = a\sin(2\pi n_1 t) + a\sin(2\pi n_2 t)$$

or,

$$y = 2a\sin\left[2\pi\left(\frac{n_1 + n_2}{2}\right)t\right]\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right]$$

[By using formula,

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)]$$

Rearranging the a bove equation, we get

$$y = 2a\cos\left[\frac{2\pi(n_1 - n_2)}{2}t\right]\sin\left[\frac{2\pi(n_1 + n_2)}{2}t\right]$$

Substituting

$$y = 2a\cos\left[\frac{2\pi(n_1 - n_2)}{2}t\right] = A \quad \text{and}$$

$\frac{n_1 - n_2}{2} = n$, we get

$$y = A\sin(2\pi nt)$$

This is the equation of a progressive wave having frequency n and amplitude A. The frequency n is the mean of the frequencies n_1 and n_2 of arriving waves while the amplitude A varies periodically with time.

The intensity of sound is proportional to the square of the amplitude. Hence the resultant intensity will be maximum when the amplitude is maximum.

For maximum amplitude (waxing),

$$A = \pm 2a$$

$$\therefore 2a\cos\left[\frac{2\pi(n_1 - n_2)}{2}\right]t = \pm 2a$$

$$\text{or, } \cos\left[\frac{2\pi(n_1 - n_2)}{2}\right]t = \pm 1$$

$$\text{i.e.}\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0, \pi, 2\pi, 3\pi, ...$$

$$\therefore t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \frac{3}{n_1 - n_2}, ...$$

Thus, the time interval between two successie maxima of sound is always $\frac{1}{n_1 - n_2}$.

Hence the period of beats is $T = \frac{1}{n_1 - n_2}$.

The number of waxing heard per second is the reciprocal of period of waxing.

$\therefore$ frequency of beats, $N = n_1 - n_2$

The intensity of sound will be minimum when amplitude is zero (waning):

For minimum a mplitude, $A = 0$,

$$\therefore 2a\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0$$

$$\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right]=0$$

$$\therefore\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right]=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},......$$

$$\therefore t=\frac{1}{2(n_1-n_2)},\frac{3}{2(n_1-n_2)},\frac{5}{2(n_1-n_2)},$$

Therefore time interval between two successive minima is also $\frac{1}{(n_1-n_2)}$, which expected.

b) Two sound waves of the wavelength 1m and 1.01 m produce 6 beats in two second when sounded together in air. Find the velocity of sound in air.

$\Rightarrow \lambda_1 = 1\text{m} \qquad \lambda_2 = 1.01\text{m}$

$$\text{(No of beats per second)} = \frac{6}{2} = 3$$

$$\lambda_1 < \lambda_2$$

$$n_1 > n_2$$

$$\therefore n_1 - n_2 = 3$$

Now

$$n_1 - n_2 = V\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]$$

$$3 = V\left[\frac{1}{1}-\frac{1}{1.01}\right]$$

$$\therefore 3 = V\left[\frac{1.01-1}{1.01}\right] = V\left[\frac{0.01}{1.01}\right]$$

$$\therefore 3 = V\left[9.9\times10^{-3}\right].$$

$$V = \frac{3}{9.9}\times10^{+3}$$

$$V = 0.303\times10^{3}$$

$$\boxed{V = 303\text{m/s}}$$

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