SHIKSHA CLASSES

BOARD ANSWER PAPER

Subject : Physics **Topic: 5. Oscillation** **Total Marks : 20**

Class : XII

Q. 1.(a): Select and write the most appropriate answer from given alternatives in each sub-question **[4]**

1. **Which of the following quantities of a simple harmonic motion does not vary sinsoidally with time?**

Ans : d) Total Energy

2. **Period of simple pendulum will be doubled if**

Ans : a) Its length is increased to four times.

3. **Presence of a damping in an oscillator.**

Ans : a) Reduces the amplitude frequency.

4. **A mass m attached to a spring of force constant k executes S.H.M. given by the equation. x = 0.5 cos (0.8 t – 0.4) metre. For this motion the ratio $\frac{k}{m}$ (in SI unit) is**

Ans : c) 0.64

Q.1. (b) : Very short answer type questions. **[2]**

i) State the differential equation of S.H.M.

Ans : Differential equation of S.H.M. : In a linear S.H.M., the force is directed towards the mean position and its magnitude is directly proportional to the displacement of the body from mean position.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\therefore \frac{d^2x}{dt^2} = -\omega^2 x$$

$$acc^n = -\omega^2 x$$

ii) Define phase of S.H.M. and epoch of a particle performing S.H.M.

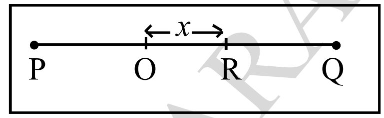
Ans : Phase of S. H. M. : *A physical quantity which gives the state of oscillation of a particle performing S.H.M. at any instant is called phase of S.H.M.*

Epoch of S.H.M. : *A physical quantity which gives the state of oscillation of a particle performing S.H.M. at time (t) =O is called epoch of S.H.M.*

Q.2. Attempt any three. **[6]**

1. Obtain the differential equation of a particle performing linear S.H.M.

Ans : Consider a particle of mass (m) performing linear S.H.M. along a straight line PQ about the mean position O (fig). Let at a time t, the particle is at a distance of x from the mean position, at R. The velocity of the particle $v = \frac{dx}{dt}$



$\therefore$ its acceleration $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

According to Newton's second law of motion, the force acting on the particle is

Force = mass × acceleration

$$F = m.\frac{d^2x}{dt^2} \quad .... \quad (i)$$

The particle performing linear SHM satisfies the following two conditions. The force acting on the particle is directly proportional to the displacement from the mean position.

The force acting on the particle is directed towards the mean position.

Both these conditions can be expressed mathematically

as $F = -kx$ .... (ii)

Where k is force constant from (i) and (ii)

$$m\frac{d^2x}{dt^2} = -kx \qquad \therefore m\frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad .... (iii)$$

where $\frac{k}{m} = \omega^2 =$ constant

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad .... (iv)$$

Equation (iii) and (iv) represent different equation of linear S.H.M.

2. State laws of simple pendulum.

Ans : **i) Law of length :** The period of a simple pendulum is directly proportional to square root of its length . i.e. $T \alpha \sqrt{l}$

ii) Law of acceleration due to gravity :

The period of simple pendulum is inversely proportional to square root of acceleration due to gravity.

i.e. $T \alpha \frac{1}{\sqrt{g}}$

iii) Law of mass : The period of simple pendulum does not depend upon its mass.

iv) Law of isochronous : The period of simple pendulum does not depend upon its amplitude.

Second's Pendulum : A simple pendulum whose period is two second is called second's pendulum.

3. Calculate the length of a seconds pendulum at a place where g = 9.8 m/s².

Ans : For second's pendulum , the length is given by

$$l = \frac{g}{\pi^2}$$

$$\therefore l = \frac{9.8}{9.86} = 0.99 \approx 1m$$

4. A particle performing linear SHM has maximum velocity of 20 cm/s and maximum acceleration of 80 cm/s². Find the amplitude and periods of oscillation.

Ans :

$$\frac{acc^n_{max}}{V_{max}} = \frac{a\omega^2}{a\omega} = \omega$$

$$\frac{80}{40} = \omega$$

$$\therefore \omega = 4\ rad/\sec$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4}$$

$$T = \frac{3.14}{2} = 1.57 \text{ sec}$$

$$V_{max} = a\omega$$

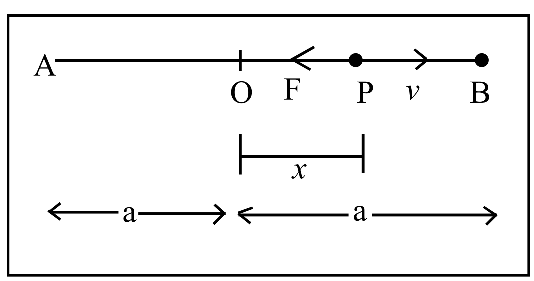
$$a = \frac{V_{max}}{\omega}$$

$$= \frac{20}{4} = 5 \text{ cm}$$

Q. 3. Attempt any one of following . [3]

1. Obtain expressions for the kinetic energy, potential energy and total energy of a particle performing linear S.H.M.

Ans : Consider a particle of mass 'm' performing linear S.H.M. of amplitude a along a line AOB with O as the mean position. Let at a time t the particle is at a point P at a distance of x from the mean position O.



Kinetic Energy in S.H.M.

The velocity of the particle when it is at P at a distance x from mean position O is given by

$$\therefore \quad V = \omega\sqrt{a^2 - x^2}$$

$$V^2 = \omega^2 (a^2 - x^2)$$

$\therefore$ Kinetic energy = $\frac{1}{2}mv^2$

$$= \frac{1}{2}m\left[\omega^2(a^2 - x^2)\right]$$

$$\text{KE} = \frac{1}{2}m\omega^2(a^2 - x^2) \quad ...(1)$$

Thus the K.E. of a particle performing linear S.H.M. depends on (1) mass (2) angular frequency (3) square of amplitude (4) square of distance from mean position.

KE at mean position i.e. at $x = 0$

$$\text{KE} = \frac{1}{2}m\omega^2 a^2 \quad [\text{from eq}^n\ (1)]$$

KE at the extreme position i.e. $x = \pm a$

$$\text{KE} = \frac{1}{2}m\omega^2(a^2 - a^2) = 0$$

$$\text{KE} = \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}k(a^2 - x^2)$$

$$\left(\because \omega^2 = \frac{k}{m}\right)$$

Potential Energy of particle performing linear S.H.M.

The workdone by external agent against restoring force in displacing a particle from its mean position to a given point on the path is called potential energy of particle performing linear S.H.M.

The restoring force (F) acting on the particle of mass 'm' at P when it is at a distance of x from O is given by.

$$\text{F} = -kx.$$

The magnitude can be assumed to be constant over a very small distance dx against the direction of the force F. The work done (dw) on the particle during this displacement is given by $dw = -\text{F}dx$ ... negative sign indicates displacement and force are in the opposite direction.

$$\text{dw} = -(-\text{k}x).\,\text{d}x = \text{k}x\,\text{d}x$$

The total work done (W) on the particle when it moves from O to P is given by

$$W = \int_0^x kx\ dx = k\int_0^x x\ dx$$

$$= k\left[\frac{x^2}{2}\right]_0^2 = k\left[\frac{x^2}{2} - 0\right]$$

W = $\frac{1}{2}kx^2$. This work done is stored in the particle as potential energy.

$\therefore$ Potential energy of the patricle at

$$P = \frac{1}{2}kx^2$$

$$\omega^2 = k/m \quad \therefore k = \omega^2 m$$

$$PE = \frac{1}{2}m\omega^2 x^2 \qquad \text{........ (2)}$$

Thus the P.E. of the particle performing linear S.H.M. depends on (1) mass of the particle (2) square of the angular frequency (3) square of the displacement.

a) PE at mean position i.e. at $x = 0$,

$$PE = \frac{1}{2}m\omega^2 x^2 = 0$$

b) At extreme position $x = \pm a$

$$\therefore \quad PE = \frac{1}{2}m\omega^2 a^2$$

Total Energy of a particle performing linear S.H.M.

Total energy of the particle is the sum of its kinetic and potential energy.

T.E. = K.E. + P.E.

$$= \frac{1}{2}m\omega^2(a^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$\text{T.E.} = \frac{1}{2}m\omega^2 a^2$$

This is the expression for total energy of particle performing linear S.H.M. Since m, w and a are all constants for a given S.H.M. the T.E. of particle performing S.H.M. remains constant it is independent of displacement *x*.

If we put $\omega = 2\pi n$

$$\text{T.E.} = \frac{1}{2}m(2\pi n)^2 . \, a^2$$

$$= 2\pi^2 m n^2 a^2$$

$\therefore$ T.E. is directly proportional it (i) mass of the particle (ii) square of amplitude (iii) square of frequency of oscillation.

2. **The displacement of a S.H.M. is given by,** $x = 12\sin(0.8\pi t) + 5\cos(0.8\pi t)$cm. **Find the amplitude, period, frequency and initial phase of S.H.M.**

Ans : Given : $x = 12\sin(0.8\pi t) + 5\cos(0.8\pi t)$

Comparing above equation

$$x = a\sin(\omega t) b\cos(\omega t)$$

$$x = a\sin\omega t.\cos\phi A\sin\phi.\cos\omega t$$

$$A\cos\phi = 12 \qquad A\sin\phi = 5$$

$$A^2\cos^2\phi + A^2\sin^2\phi = 12^2 + 5^2$$

$$A^2(\cos^2\phi + A^2\sin^2\phi) = 144 + 25$$

$$A^2 = 169 \qquad \boxed{A = 13}$$

$$\omega t = 0.8\pi t$$

$$\omega = 0.8\pi$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.8\pi} = \frac{2}{0.8} =$$

$$\boxed{T = 2.5 \text{ second}}$$

$$A\cos\phi = 12$$

$$A\sin\phi = 5$$

$$\frac{A\sin\phi}{A\cos\phi} = \frac{5}{12}$$

$$\tan\phi = \frac{5}{12}$$

$$\phi = \tan^{-1}(0.4166)$$

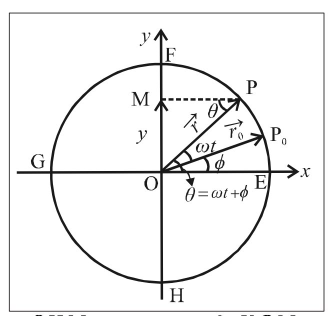
$$\boxed{\phi = 22^0 61^1}$$

Q. 4. Attempt any one **[5]**

1. **Show that a linear S.H.M. is the projection of a U.C.M. along any of its diameter.**

   In summer season, a pendulum clock is regulated as a second's pendulum and it keeps correct time. During winter, the length of the pendulum decreases by 1%. How much will the clock gain or lose in one day. (g = 9.8 m/s²)

Ans : There is basic relation between S.H.M. and U.C.M. that is very useful in understanding S.H.M. For an object performing U.C.M. the projection of its motion along any diameter of its path executes S.H.M.



S.H.M. as projection of a U.C.M.

Consider particle 'P' is moving along the circumference of circle o radius 'a' with constant angular speed of ω in anticlockwise direction as shown in figure.

Particle P along circumference of circle has its projection particle on diameter AB at point M. Particle P is called reference particle and the circle on which it moves, its projection moves back and forth along the horizontal diameter, AB.

The x–component of the displacement of P is always same as displacement of M, the x–component of the velocity of P is always same as velocity of M and the x–component of the acceleration of M.

Suppose that particle P starts from initial position with initial phase α (angle between radius OP and the x – axis at the time $t = 0$) In time t the angle between OP and x - axis is $(\omega t + \alpha)$ as particle P moving with constant angular velocity (ω) as shown in figure.

$$\cos(\omega t + \alpha) = \frac{x}{a}$$

$$\therefore x = a\cos(\omega t + \alpha) \quad ....(1)$$

This is the expression for displacement of particle M at time t.

As velocity of the particle is the time rate of change of displacement then we have

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}[a\cos(\omega t + \alpha)]$$

$$\therefore \upsilon = -a\omega\sin(\omega t + \alpha) \quad .....(2)$$

As acceleration of particle is the time rate of change of velocity, we have

$$a = \frac{dx}{dt} = \frac{d}{dt}[-a\omega\sin(\omega t + \alpha)]$$

$$\therefore a = -a\omega^2\cos(\omega t + \alpha)$$

$$\therefore \ a = \omega^2 x$$

It shows that acceleration of particle M is directly proportional to its displacement and its direction is opposite to that of displament. Thus particle M performs simple harmonic motion but M is projection of particle performing U.C.M. hence S.H.M. is projection of U.C.M. along a diameter, of circle

We need to calculate time lost (ΔT) in seconds per day (T = 24 × 60 × 60 × = 86400s). We know that

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \ell = \frac{T^2 g}{4\pi^2} \quad ---(I)$$

$$\Rightarrow \Delta\ell = \frac{2T\Delta T g}{4\pi^2} \quad ---(II)$$

now divide (II) by (I), we have

$$\Rightarrow \frac{\Delta\ell}{\ell} = \frac{2\Delta T}{T}$$

$$\Rightarrow \Delta T = \frac{T}{2}\left(\frac{\Delta\ell}{\ell}\right)$$

$$\Rightarrow \Delta T = \frac{86400}{2}(-0.01)s = -432s$$

OR

2. **State the differential equation of S.H.M. and hence obtain an expression for the velocity of a particle performing S.H.M. and discuss its minimum and maximum value.**

A particle performs S.H.M. of period $2\pi/\sqrt{3}$ sec along a path 4 cm long. Calculate the displacement of the particle at which its velocity is numerically equal to acceleration.

Ans : **Differential Equation of S.H.M. :** In a linear S.H.M., the force is directed towards the mean position and its magnitude is directly proportional to the displacement of the body from mean position.

where k is force constant and x is displacement from the mean position.

According to Newton's second law of motion,

f = ma $\quad \therefore$ ma = - kx

Expressions for the acceleration, velocity and displacement of a particle performing S.H.M. by solving the differential equation of S.H.M. in terms of displacement x and time t.

$$\therefore \frac{d^2x}{dt^2} = -\omega^2 x \quad ---(i)$$

But $a = \frac{d^2x}{dt^2}$ is the acceleration of the particle performing S.H.M.

$$a = -\omega^2 x \qquad ---(ii)$$

This is the expression for acceleration in terms of displacement x.

Form (i), we have $\frac{d^2x}{dt^2} = -\omega^2 x$

$$\therefore \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\omega^2 x$$

$$\therefore \frac{dv}{dt} = -\omega^2 x$$

$$\therefore \frac{dv}{dx}\frac{dx}{dt} = -\omega^2 x$$

$$\therefore v\frac{dv}{dx} = -\omega^2 x$$

$$\therefore v\,dv = -\omega^2 x\,dx$$

Integrating both the sides, we get

$$\int v\,dv = -\omega^2 \int x\,dx$$

$$\therefore \frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C \qquad ---(iii)$$

where C is the constant of integration.
Let A be the maximum displacement (amplitude) of the particle in S.H.M.
When the particle is at the extreme position, velocity (v) is zero.
Thus, at x $x = \pm A, v = 0$
Substituting in Eq. (ii), we get

$$0 = -\frac{\omega^2 A^2}{2} + C$$

$$\therefore C = +\frac{\omega^2 A^2}{2} \quad ---(iv)$$

Using C in Eq. (iii), we get

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$$\therefore v^2 = \omega^2 (A^2 - x^2)$$

$$\therefore v = \pm\omega\sqrt{A^2 - x^2} \qquad ---(v)$$

This is the expression for the velocity of a particle performing linear S.H.M. in terms of displacement x.
The magnitude of velocity of the particle performing S.H.M. is $v = \pm\omega\sqrt{A^2 - x^2}$
At the mean position, $x = 0 \therefore v_{min} = \pm A\omega$.
Thus, the velocity of the particle in S.H.M. is maximum at the mean postion.
At the extreme position, $x = \pm A \therefore v_{min} = 0$.
Thus, the velovity of the particle in S.H.M is minimum at the extrem position.

$$\omega\sqrt{A^2 - x^2} = \omega^2 x \therefore \sqrt{A^2 - x^2} = \omega x \quad ...(i)$$

$$T = \frac{2\pi}{\sqrt{3}} \;\&\; \omega = \frac{2\pi}{T} = \sqrt{3}$$

Substituting value ω in equation (i) we get

$$\sqrt{A^2 - x^2} = \sqrt{3}x$$

$$A = 2x$$

$$\text{As amplitude} = \frac{\text{Pathlength}}{2}$$

$$= \frac{2\text{ cm}}{2}$$

$$\boxed{x = 1\text{ cm}}$$

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