SHIKSHA CLASSES

Subject : Physics **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 2. Mechanical Properties of Fluids**

Q.1 : A) Select and write the most appropriate answer from given alternatives in each sub-question. **4M**

1) Ideal fluids are…

Ans. : d) All of above

2) What is the ratio of surface energy of 1 small drop and 1 large drop, if 1000 small drop combined to form 1 large drop.

Ans. : a) 1 : 100

Volume of liquid remain same i.e. volume of 1000 small drops will be equal to volume of one big drop.

$$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 R^3 = 1000r^3 R = 10r \quad \therefore \frac{r}{R} = \frac{1}{10}$$

$$\frac{\text{Surface energy of one small drop}}{\text{Surface energy of one big drop}} = \frac{4\pi r^2 T}{4\pi R^2 T} = \frac{1}{100}$$

3) The excess of pressure inside a soap bubble than that of the outer pressure is

Ans. : b) 4T/r

4) Two capillary tubes of radii 0.3 cm and 0.6 cm are dipped in the same liquid. The ratio of heights through which the liquid will rise in the tubes is -

Ans. : b) 2:1

$$\Rightarrow h_1 r_1 = h_2 r_2$$

$$\frac{h_1}{h_2} = \frac{r_2}{r_1} = \frac{\overset{2}{\cancel{0.6\ cm}}}{\underset{1}{\cancel{0.3\ cm}}} = \frac{2}{1}$$

Q.1 : B) Very short answers type questions. **2M**

1) Define surface tension.

Ans. : Surface tension T is defined as, the tangential force acting per unit length on both side of an imaginary line drawn on the free surface of liquid.

$$T = \frac{F}{L}$$

SI unit of surface tension is N/m. Its Dimension are, $[L^0M^1T^{-2}]$.

2) State Newton's Law of Viscosity.

Ans. : According to Newton's law of viscosity, for a steamline flow, viscous force (*f*) acting on any layer is directly proportional to the area (A) of the layer and the velocity gradient (dv/dx) i.e.,

$$f \alpha A\left(\frac{dv}{dx}\right)$$

$$\therefore f = \eta A\left(\frac{dv}{dx}\right)$$

where η is a constant, called coefficient of viscosity of the liquid.

$$\eta = \frac{f}{A\left(\frac{dv}{dx}\right)}$$

Q.2 : Attempt any THREE. **6M**

1) Calculate the work done in increasing the radius of soap bubble in air from 1cm to 2cm. The surface tension of a soap bubble is 30 dyne/cm.

Ans. : T = 30 dyne / cm

to find : work done = ?

work done $= TdA = T \times 2 \times 4\pi(r_2^2 - r_1^2)$

$$= T \times 8\pi(4-1) = 24\pi T$$

$$= 24 \times 3.14 \times 30 = 2260.8\ erg.$$

$$\boxed{W = 2260.8\ erg}$$

2) State Pascal's law. State its applications.

Ans. : Pascal's law states that the pressure applied at any point of an enclosed fluid at rest is transmitted equally and undiminished to every point of the fluid and also on the walls of the container, provided the effect of gravity is neglected.

Applications of Pascal's Law :

i) Hydraulic lift ii) Hydraulic brakes

3) A liquid rises to a height of 5 cm in a glass capillary of radius 0.02 cm. What will be the height of the same liquid in a glass capillary of radius 0.04cm?

Ans. : Given : h_1 = 5cm, r_1 = 0.02 cm, r_2 = 0.04 cm

To find : h_2 = ?

$$h_1 r_1 = h_2 r_2$$

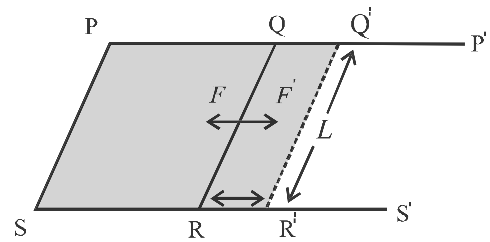
$$h_2 = \frac{h_1 r_1}{r_2} = \frac{5 \times 0.02}{0.04}$$

$$h_2 = 2.5\text{cm}$$

4) **Define surface energy. Obtain the relation between surface tension and surface energy.**

Ans. : Surface molecules possess extra potential energy as compared to the molecules inside the liquid. The extra energy of the molecules in the surface layer is called the surface energy of the liquid.

Consider a rectangular frame of wire P'PSS'. It is fitted with a movable arm QR as shown in fig. This frame is dipped in a soap solution and then taken out. A film of soap soluion will be formed within the boundaries PQRS of the frame.



Surface energy of a liquid

Each arm of the frame experiences an inward force due to the film. Under the action of this force, the movable arm QR moves towards side PS so as to decrease the area of the film. If the length of QR is L, then this inward force F acting on it is given by

$F = (T) \times (2\,L)$

Since the film has two surfaces, the upper surface and the lower surface, the total length over which surface tension acts on QR is 2L. Imagine an external force F' (equal and opposite of F) applied isothermally (gradually and at constant temperature, to the arm QR, so that it pulls the arm away and tries to increases the surface area of the film. The are QR movs to Q'R' through a distance dx. Therefore, the work done against F, the force due to surface tension, is given by

$dw = F'\,dx$

Using Eq.

$dw = T\,(2\,L\,dx)$

But 2 Ldx = dA, increase in area of the two surfaces of the film. Therefore, dw = T(dA). This work done in stretching the film is stored in the area dA of the film as its potential energy. This energy is called surface energy.

$\therefore$ Surface energy = T (dA)

Thus, surface tension is also equal to the surface energy per unit area.

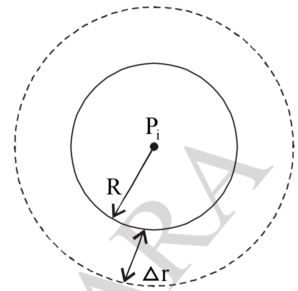
Q.3 : Attempt any one of following. **3M**

1) **Derive an expression of excess pressure inside a liquid drop.**

Ans. : Consider a spherical drop as shown in. Let P_i be pressure inside the drop and P_o be the pressure out side it. As the drop is spherical in shape. The pressure, P_i, inside the drop is greater than P_o, the pressure outside. Therefore, the excess pressure inside the drop is P_i-P_o.

Let the radius of the drop increase from r to r + Δr, where Δr is very small, so that the pressure inside the drop remains almost constant. Let the initial surface area of the drop be $A_1 = 4\pi r_1^2$ and the final surface area of the drop be

$A_2 = 4\pi(r + \Delta r)^2$



$\therefore A_2 = 4\pi(r^2 + 2r\Delta r + \Delta r^2)$

$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r + 4\pi\Delta r^2$

As Δr is very small, Δr^2 can be neglected,

$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r$

Thus, increase in the surface area of the drop is $dA = A_2 - A_1 = 8\pi r\Delta r$ ----(i)

work done in increasing the surface area by dA is stored as excess surface energy

$\therefore dW = TdA = T(8\pi r\Delta r)$ ----(ii)

This work done is also equal to the product of the force F which cause increase in the area of the bubble and the displacement Δr which is the increase in the radius of the bubble

$\therefore dW = F\Delta r$ ----(iii)

The excess force is given by,

(Excess pressure) × (Surface area)

$\therefore F = (P_i - P_o)4\pi r^2$ ----(iv)

Equating Eq. (ii) and eq. (ii), we get

$T(8\pi r\Delta r) = (P_i - P_o)4\pi r^2\Delta r$

$\therefore (P_i - P_o) = \frac{2T}{r}$ ----(v)

This equation gives the excess pressure inside a drop. this is called laplace's law of a spherical membrane.

2) **Calculate the amount of energy evolved when 125 droplets of mercury each of radius 0.05 mm, combine to form one droplet. The surface tension of mercury is 50 x 10^{-2} N/m.**

Ans. : **Given :** R = radius of big drop P_1r = radius of small drop, T = 50 × 10^{-2} N/m, r = 0.05 × 10^{-3}m,

To find : Energy = ?

Solution : volume of done drop = volume of 343 droplets

$\therefore \frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$

$\therefore R^3 = 125 r^3$

$\boxed{\therefore R = 5r}$

Energy evolved = surface tension × change in area

$$= TdA = T\times\left[125\times4\pi r^2 - 4\pi R^2\right]$$

$$= T\times\left[125\times4\pi r^2 - 4\pi\left(5r\right)^2\right]$$

$$= T = 4\pi r^2\left[125-25\right] = T\times4\pi r^2\times100$$

$$= 50\times10^{-2}\times4\pi = (0.05\times10^{-3})^2\times100$$

$$= 50\times10^{-2}\times4\pi = 0.0025\times10^{-6}\times100$$

$$= 200\times10^{-2}\times\pi\times100\times25\times10^{-4}\times10^{-6}$$

$$= 2\times\pi\times25\times100\times10^{2-2-4-6}$$

$$= 50\times3.14\times100\times10^{-10}$$

$$= 157\times10^{-8}$$

$$= 1.57\times10^{-10}\,J$$

Q.4 : Attempt any one. **5M**

i) Derive an expression for capillary rise by using pressure difference method.

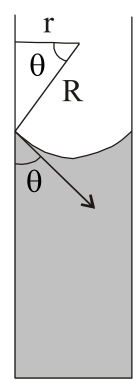
ii) Water column inside a glass capillary tube is 4.5 cm long. Mercury surface in the same tube is depressed by 1.25 cm. compare the surface tension of the mercury with that of water.

Ans. : i) The pressure due to the liquid (water) column of height h must be equal to the pressure difference 2T/R due to the concavity.

$$\therefore h\rho g = \frac{2T}{R} \quad \text{---- (i)}$$

Where, ρ is the density of the liquid and g is acceleration due to gravity.

Let r be the radius of the capillary tube and θ be the angle of contact of the liquid as shown in fig. (a).



(a) Force acting at The point of contact

Then radius of curvature R of the meniscus is given by $R = \frac{r}{\cos\theta}$

$$\therefore h\rho g = \frac{2T\cos\theta}{r}$$

$$\therefore h = \frac{2T\cos\theta}{r\rho g} \quad \text{----(ii)}$$

The above equation gives the expression for capillary rise (or fall) for a liquid. Narrower the tube, the greater is the height to which the liquid rises (or falls).

If the capillary tube is held vertical in a liquid that has a convex meniscus, then the angle of contact θ is obtuse. Therefore, $\cos\theta$ is negative and so is h. This means that the liquid will suffer capillary fall or depression.

Given : $h_w = 4.5$ cm, $h_m = -1.25$ cm

$\rho_w = 10^3$ kg/m³, $\rho_m = 13.6 \times 10^3$ kg/m³

$\theta_w = 0^0 \ \theta_m = 135^0$

To find = T_w = ?

Solution : $T = \dfrac{hr\rho g}{2\cos\theta}$

$$T_w = \frac{h_w r \rho_w g}{2\cos\theta}$$

$$T_m = \frac{h_m r \rho_m g}{2\cos\theta_m}$$

$$\therefore \frac{T_m}{T_w} = \frac{h_m r \rho_m g}{2\cos\theta_m} \times \frac{2\cos\theta_w}{h_w r \rho_w g}$$

$$\frac{T_m}{T_w} = \frac{h_m \rho_m \cos\theta_w}{h_w \rho_w \cos\theta_m}$$

$$\frac{T_m}{T_w} = \frac{-1.25\times10^{-2}\times13.6\times10^3\times\cos 0}{4.5\times10^{-2}\times10^3\times\cos 135}$$

$$= \frac{-1.25\times13.6}{4.5\times(-0.707)} = \frac{1.25\times13.6}{4.5\times0.707}$$

$$\frac{T_m}{T_w} = 5.343$$

$$\boxed{\therefore T_m : T_w = 5.343 : 1}$$

OR

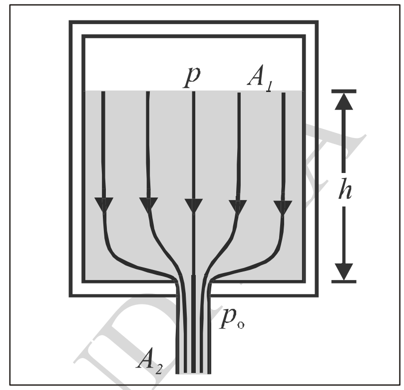
State and prove Torricelli's theorem.

What terminal velocity will aluminum sphere of radius in 1mm falling through water at 20⁰C and specific gravity of aluminum 2.7? $(\eta_{water} = 8\times10^{-4}\,\text{Ns/m}^2)$

Ans.: The word efflux means fluid out flow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body.

Consider a liquid of density 'ρ' filled in a tank of large cross-sectional area A_1 having an orifice of cross-sectional area A_2 at the bottom as shown in Fig. Let $A_2 << A_1$. The liquid flows out of the tank through the orifice. Let v_1 and v_2 be the speeds of the liquid at A_1 and A_2 respectively. As both, inlet and outlet, are exposed to the atmosphere, the pressure at these position equals the atmosphere pressure p_0. If the height of the free surface above the orifice is h, Bernoulli's equation gives us,

$$p_0 + \frac{1}{2}\rho v_1^2 + \rho g h = p_0 + \frac{1}{2}\rho v_2^2 \qquad \text{---(i)}$$



Efflux of fluid from an orifice.

Using equation the of continuity we can write,

$$v_1 = \frac{A_2}{A_1} v_2$$

Substituting v_1 in Eq. (i) we get,

$$\frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 + \rho g h = \frac{1}{2}\rho v_2^2$$

$$\left(\frac{A_2}{A_1}\right)^2 v_2^2 + 2gh = v_2^2$$

$$2gh = v_2^2 - \left(\frac{A_2}{A_1}\right)^2 v_2^2$$

$$\therefore \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] v_2^2 = 2gh$$

If $A_2 << A_1$, the above equation reduces to,

$$v_2 = \sqrt{2gh} \qquad \text{---(ii)}$$

This is the equation of the speed of a liquid flowing out through an orifice at a depth 'h' below the free surface. It is the same as that of a particle falling freely through the height 'h' under gravity.

Here, r = 1 mm = 10^{-3} m

specific gravity of Al = 2.7

Density of Al, ρ = 2.7 × 10^3 kg/m^3

Density of water, σ = 10^3 kg/m^3

$$\eta = 8\times10^{-4}\,\mathrm{Ns/m^2},\ g = 9.8\,\mathrm{m/s^2}$$

$$V_T = \frac{2r^2(\rho-\sigma)g}{9\eta}$$

$$\therefore V_T = \frac{2(10^{-3})^2\times(2.7\times10^3 - 10^3)\times9.8}{9\times8\times10^{-4}}$$

$$\therefore V_T = \frac{2\times10^{-6}\times1.7\times10^3\times9.8}{9\times8\times10^{-4}}$$

$$\boxed{V_T = 4.627\ \mathrm{m/s}}$$

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