SHIKSHA CLASSES

Subject : Physics **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 1. Rotational Dynamics**

Q.1 : A) Select and write the most appropriate answer from given alternatives in each sub-question. **4M**

1) A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom.

Ans. : a) $\frac{5}{7} g \sin\theta$

$$a = \frac{g \sin\theta}{1+\frac{k^2}{R^2}} = \frac{g \sin\theta}{1+\frac{2}{5}} = \frac{5}{7} g \sin\theta$$

2) A disc and sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of two objects gets to the bottom of the plane first…

Ans. : b) Sphere

$$a = \frac{g \sin\theta}{1+\frac{k^2}{R^2}}$$

$$\text{disc} = \frac{k^2}{R^2} = \frac{1}{2} = 0.5$$

$$\text{k or sphere} \frac{k^2}{R^2} = \frac{2}{5} = 0.4$$

$\therefore$ sphere > disc

$\therefore$ sphere reacher first

3) What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

Ans. : d) $\sqrt{5gR}$

When minimum speed of body is $\sqrt{5gR}$. Then no matter from where is envers the loog, is will complete full vertical loop.

4) A cyclist goes round a circular path of circumference 34.3 m in 1 sec. the angle made by him, with the vertical, will be.

Ans.: a) 45 degree

$$2\pi r = 34.3$$

$$r = \frac{34.3}{2\pi} \;\&\; v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$$

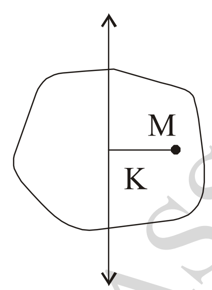
$$\tan\theta = \frac{v^2}{rg} = \frac{\left(\frac{34.3}{\sqrt{22}}\right)^2}{\frac{34.3}{2\pi}\times 9.8} \approx 1$$

$$\theta = \tan^{-1}(1) = 45^0$$

Q.1 :B) Very short answers type questions. 2M

1) Define radius of gyration?

Ans.: **Radius of gyration :** Radius of gyration of a body about an unit of rotation is the distance between the units of rotation and a point, at which the whole mass of the body is supposed to be concentrated so as to have the same moment of inertia as that of the body about the same axis of rotation.



$$I = mk^2$$

$$k = \sqrt{I/m}$$

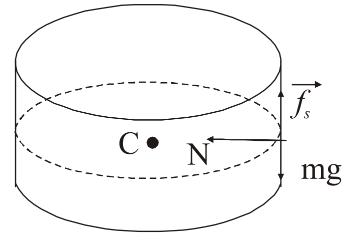
2) State two examples of the principle of conservation of angular momentum.

Ans.: Ballet dance, acrobat in a circus, sports like ice skating, diving in a swimming pool, etc., these are examples of the principle of conservation of angular momentum.

Q.2 : Attempt any THREE. 6M

1) Derive an expression for minimum speed at which body remains in contact with wall of well of death.

Ans.:



A motor cyclist, is to under take horizontal circles inside the cylindrical wall of radius r. the forces acting on the vehicle (assumed to be a point) are

i) Normal reaction 'N' acting horizontally and towards the centre.

ii) Weight 'mg' acting vertically down ward.

iii) Force of static friction (f_s) acting vertically upwards between vertical wall and the tyres it is static friction because to prevent downward Slipping and it magnitude equal to 'mg' and its direction only upward force.

Normal reaction N is a resultant centripetal force (that balance centrifugal force).

$$\therefore N = mr\omega^2 = \frac{mv^2}{r} \;\&\; mg = f_s$$

f_s static friction is always less than or equal to $\mu_s N$

$$\therefore f_s \le \mu_s N$$

$$\therefore mg \le \mu_s \left(\frac{mv^2}{r}\right)$$

$$\therefore g \ge \frac{\mu_s v^2}{2}$$

$$\therefore v^2 \ge \frac{rg}{\mu_s}$$

$$\boxed{\therefore V_{\min} = \sqrt{\frac{rg}{\mu_s}}}$$

2) Show that M.I. of a rod about an axis passing through one end and perpendicular to its length is $\frac{ML^2}{3}$.

Ans : Given,

Mass of rod = M

Length of Rod = L

Distance of rods edge from center $= \frac{L}{2}$

Moment of inertia of rod at center $I_c = \frac{1}{12} ML^2$

For moment of inertia at edge apply parallel axis

theorem. $I = I_c + M\left(\frac{L}{2}\right)^2$

$$I = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2$$

moment of inertia at edge is $\frac{1}{3} ML^2$.

3) What is the ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes?

Ans.: Radius of Gyration of circular disc

$$K_{disc} = \frac{R}{\sqrt{2}}$$

radius of Gyration of circular ring

$$K_{Ring} = R$$

$\therefore$ Ratio of radii of gyration of disc to ring is

$$\therefore \quad \frac{K_{disc}}{K_{Ring}} = \frac{\frac{R}{\sqrt{2}}}{R} = \frac{1}{\sqrt{2}}$$

4) A curve of radius 50m is banked at 30^0. Will a car travelling at 60 km/hr along the curve be safe or unsafe?

Ans.: **Given :** r = 50m $\theta = 30^0$

$$v = 60\ km/hr = 60 \times \frac{50}{3} = 16.66\ m/s.$$

maximum speed of the vehicle

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{50 \times 9.8 \times \tan 30^0}$$

$$v = \sqrt{50 \times 9.8 \times 0.5774}$$

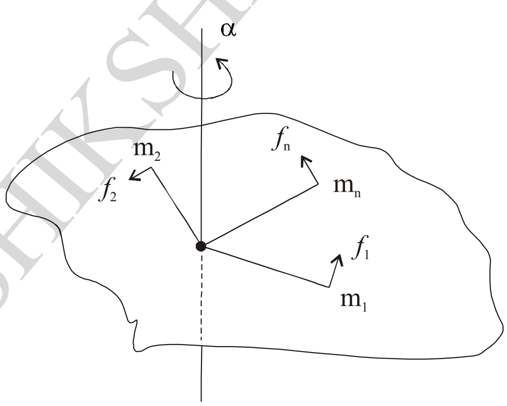
$$\boxed{v = 16.82\ m/s}$$

The actual speed of vehicle 16.66 m/s less than the maximum speed 16.82 m/s, therefore is safe for vehicle to take turn with given speed.

Q.3 : Attempt any one of following. **3M**

1) Obtain the relation between the torque and angular acceleration for a rotating body.

Ans.:



A rigid object rotating with a constant angular acceleration 'α' about an axis perpendicular to the plane of paper. Let us consider the object to be consisting of N number of particles of masses m_1, m_2, ….. m_N at respective perpendicular distances r_1, r_2, ...r_N from the axis of rotation

All these particles perform circular motion with same angular acceleration α, but with different linear (tangential) accelerations etc.

$a_1 = r_1\alpha, a_2 = r_2\alpha, a_3 = r_3\alpha....a_N = r_N\alpha$

Force experienced by the first particle is

$f_1 = m_1 a_1 = m_1 r_1 \alpha$

Thus, the torque experienced by the first particle is of magnitude $\tau_1 = f_1 r_1 = m_1 r_1^2 \alpha$

Similarly, $\tau_2 = m_2 r_2^2 \alpha, = \tau_3 = m_3 r_3^2 \alpha........\tau_N = m_N r_N^2 \alpha$

If the rotation is restricted to a single plane, directions of all these torques are the same, and along the axis. Magnitude of the resultant torque is then given by

$$\tau = \tau_1 + \tau_2 + ... + \tau_N$$

$$= \left(m_1 r_1^2 + m_2 r_2^2 ... + m_N r_N^2\right)\alpha = I\alpha$$

where, is the $I = m_1 r_1^2 + m_1 r_2^2 ... + m_N r_N^2$ moment of inertia of the object about the given axis of rotation.

2) A train of mass 10^4kg rounds a curve of radius 200 m at a speed of 90 km/hr. Find the horizontal thrust on the outer rail if the track is not banked. What angle must the track be banked in order that there is no thrust on the rail?

Ans.: **Given :** m = 10^4 kg, r = 200 m

v = 90 km/hr $= 90 \times \frac{5}{18} = 25 m/s$

Thrust(F) = ?, $\theta = ?$ (for no thrust)

Horizontal thrust = centrifugal force

$$F = \frac{mv^2}{r} = \frac{10^4 \times (25)^2}{200}$$

$$F = \frac{1000 \times 625}{200} = 31250 N$$

$$f = 3.125 \times 10^4 N$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\theta = \tan^{-1}\left(\frac{25^2}{200 \times 9.8}\right) = \tan^{-1}\left(\frac{25}{8 \times 9.8}\right)$$

$$\theta = \tan^{-1}(0.3189)$$

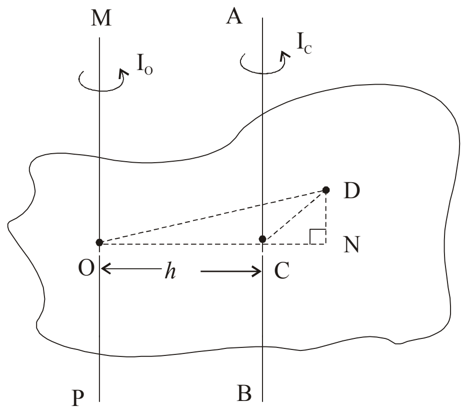
$$\boxed{\theta = 17^0 41^1}$$

Q.4 : Attempt any one. **5M**

1) State and prove the principle of parallel axes about moment of inertia.

A solid sphere of mass 1kg, radius 10 cm rolls down an inclined plane of height 7m. What will be velocity of its centre as it reaches the ground level?

Ans. :



Statement : The moment of inertia (I_o) of an object about any axis is the sum of its moment of inertia (I_C) about an unit parallel to the given axis, and passing through the centre of mass and product of the mass of object and square of the distance between the two axes (Mh^2)

$I_o = I_c + Mh^2$

Consider a rigid body of mass M. let M.I. of the body about an axis passing through O is I_o.

Let C be the centre of mass. Let M.I. of the body axis passing through the centre of mass = I_c, perpendicular distance between the two axes = h.

Consider a point D at which small element of the body of mass(dm) is situated, join OD and CD. drop perpendicular DN from D on OC extended,

$$I_o = \int dm\ OD^2$$

$$I_c = \int dm\ CD^2$$

$\therefore$ From $\triangle ODN$,

$$\therefore \quad I_o = \int (OD)^2 dm = \int \left((DN)^2 + (NO)^2\right) dm$$

$$= \int \left[(DN)^2 + (NC)^2 + 2NC.CO + CO\right]^2 dm$$

$$= \int \left[(DC)^2 + 2NC\ dm + h^2 \int dm\right.$$

Now, $\int (DC)^2 dm = I_c \ \& \int dm = M$

$\therefore$ NC is the distance of a point from the centre of mass.

Any mass distribution is symmetric about the centre of mass.

$$\int NC.dm = 0$$

$$\therefore I_o = I_c + M.h^2$$

This is a mathematical form of theorem of parallel axes.

⇒ **Given :** mass (m) = 1 kg.

Radius = 10 cm.

The kinetic energy of a rolling body is

$$\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

According to law of conservation of energy

$$\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)=mgh$$

where h is the height of the inclined plane

$$\therefore v=\sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

For a solid sphere $\frac{k^2}{R^2}=\frac{2}{5}$

Substituting the given values, we get

$$v=\sqrt{\frac{2\times 10\times 7}{\left(1+\frac{2}{5}\right)}}=\sqrt{\frac{2\times 10\times 7\times 5}{7}}=10\ ms^{-1}.$$

OR

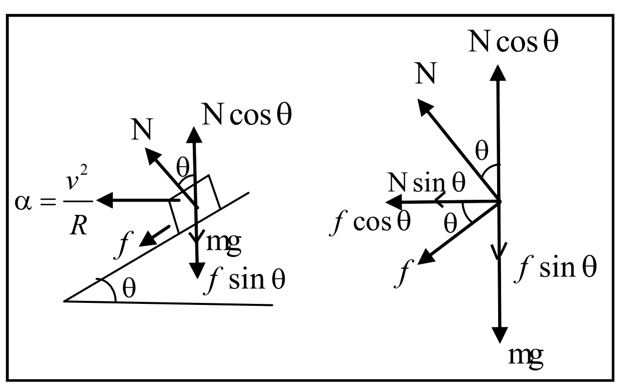
2) Obtain an expression for maximum speed along the banked road taking into account the friction between the tyres of vehicles and the road surface.

A car can just go around a curve of 10 m radius without skidding when travelling at the speed of 18 km/hr. If the road is horizontal, find the coefficient of friction between the road surface and tyres.(g=9.8 m/s²)

Ans. : Consider a vehicle of mass (M) moving on a banked road of inclination θ and radius of curvature 'r'.

mg - weight of vehicle, N - Normal reaction

F - Frictional force.



$f\cos\theta =$ Horizontal component of frictional force.

$f\sin\theta =$ Vertical component of frictional force.

$N\cos\theta =$ Vertical component of normal reaction.

$N\sin\theta =$ Horizontal component of normal reaction.

Vertical component of normal reaction N $\cos\theta$ is balanced by weight of the vehicle and component $f\sin\theta$ of frictional force.

$\therefore \quad N\cos\theta = mg + f\sin\theta$

$\therefore \quad mg = N\cos\theta - f\sin\theta$ ---(i)

Horizontal component of normal reaction $N\sin\theta$ and Horizontal component of frictional force $f\cos\theta$ provide the centripetal force.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

$$\therefore \quad \frac{mv^2}{R} = N\sin\theta + f\cos\theta \quad \text{---(ii)}$$

dividing eq^n (ii) by eq^n (i)

$$\frac{v^2}{Rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} \quad \text{---(iii)}$$

Let V_{max} - maximum speed of vehicle. Then frictional force produced at this speed should be.

$$f_m = \mu_s N$$

Put this value about eq^n (iii)

$$\frac{V_{max}^2}{Rg} = \frac{N\sin\theta + f_m\cos\theta}{N\cos\theta - f_m\sin\theta}$$

$$\frac{V_{max}^2}{Rg} = \frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta - \mu_s N\sin\theta} \quad \text{---(iv)}$$

dividing by $N\cos\theta$ in numerator and denominator of RHS.

$$\frac{V_{max}^2}{Rg} = \frac{\dfrac{N\sin\theta}{N\cos\theta} + \dfrac{\mu_s N\cos\theta}{N\cos\theta}}{\dfrac{N\cos\theta}{N\cos\theta} - \dfrac{\mu_s N\sin\theta}{N\cos\theta}}$$

$$\frac{V_{max}^2}{Rg} = \frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}$$

$$\therefore \quad V_{max}^2 = Rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)$$

$$V_{\max} = \sqrt{Rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$

This maximum velocity of the vehicle moving on a banked curved road which is not frictionless.

Given : r = 10 m

V = 18 km/hr

$= 18 \times \frac{1000}{3600}$

$\boxed{V^2 = \mu rg}$

$= 18 \times \frac{10}{36}$

$\mu = \frac{V^2}{rg} = \frac{25}{10 \times 9.8}$

$= 5\ m/s^2$

$\boxed{\mu = 0.255}$

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