Shiksha Classes Bhandara

Mathematics Topic : Limit, Continuity and Differentiability of Function MM 100

Q.1 $\lim_{x\to 0}\left[\frac{\sin[x-3]}{[x-3]}\right]$, where [.] denotes greatest integer function is –
(A) 0 (B) 1
(C) does not exist (D) sin 1

Q.2 At the end points A, B of the fixed segment of length L, lines are drawn meeting in C and making angles θ and 2θ respectively with the given segment. Let D be the foot of the altitude CD and let x represents the length of AD. The value of x as θ tends to zero i.e. $\lim_{\theta\to 0} x$ equals –
(A) L/2 (B) 2L/3
(C) 3L/4 (D) L/4

Q.3 $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{1/x}$ equals
(A) ab (B) $\sqrt{ab}$
(C) 1/ab (D) $1/\sqrt{ab}$

Q.4 If $f(x)=\begin{cases}[x], & \text{if } -3\le x<0\\ 2x+1, & \text{if } 0\le x\le 2\end{cases}$ and
g (x) = f (| x |) + | f (x) |, then in [–3, 2]
(A) $\lim_{x\to 0^+} g(x)=2$
(B) $\lim_{x\to 0^-} g(x)=0$
(C) g (x) is discontinuous at three points
(D) None of these

Q.5 If f is a periodic function, then –
(A) f ' and f " are not also periodic
(B) f ' is periodic but f " is not periodic
(C) f " is periodic but f ' is not periodic
(D) None of these

Q.6 If m, n are positive integers then
$\lim_{x\to 0}\frac{(\cos x)^{1/m}-(\cos x)^{1/n}}{x^2}=$
(A) m – n (B) $\frac{1}{n}-\frac{1}{m}$
(C) $\frac{m-n}{2mn}$ (D) None of these

Q.7 If $y = x^5 \operatorname{sgn} x$, where $\operatorname{sgn} x=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$, then f (x) is
(A) differentiable as well as continuous at x = 0
(B) continuous but not differentiable at x = 0
(C) differentiable but not continuous at x = 0
(D) neither differentiable nor continuous at x = 0

Q.8 If $\lim_{x\to 0}$ $(x^{-3}\sin 3x + ax^{-2} + b)$ exists and is equal to zero then
(A) a = – 3 & b = 9/2 (B) a = 3 & b = 9/2
(C) a = – 3 & b = – 9/2 (D) a = 3 & b = – 9/2

Q.9 The set up of points where the function
f (x) = | x – 2 | cos x is differentiable, is –
(A) (–∞, ∞) (B) (–∞, ∞)/ {2}
(C) (0, ∞) (D) None of these

Q.10 Evaluate $\lim_{x\to\infty}\frac{\sqrt{1+\sin 3x}-1}{\text{in}(1+\tan 2x)}$
(A) 1/7 (B) 2/7
(C) 5/7 (D) 3/4

Q.11 If 4x + 3 | y | = 5y, then y as a function of x is –
(A) differentiable at x = 0 (B) continuous at x = 0
(C) $\frac{dy}{dx}=2$ for all x (D) None of these

Q.12 Let f be a function satisfying f (x + y) = f (x) + f (y) and f (x) = x^3 ϕ (x) for all x and y, where ϕ (x) is a continuous function then f ' (x) is equal to –
(A) g (0) (B) g ' (x)
(C) 0 (D) None of these

Q.13 Function $f(x)=\begin{cases}-x & \text{if } x<0\\ x^2 & \text{if } 0\le x\le 1\\ x^3-x+1 & \text{if } x>1\end{cases}$
is differentiable at -
(A) x = 0 but not at x = 1 (B) x = 1 but not at x = 0
(C) x = 0 and x = 1 both (D) neither x = 0 nor x = 1

Q.14 $\lim_{n\to\infty}\cos\left(\pi\sqrt{n^2+n}\right)$ when n is an integer :
(A) is equal to 1 (B) is equal to – 1
(C) is equal to zero (D) does not exist

Q.15 If $f(x)=(-1)^{[x^3]}$, where [.] denotes the greatest integer function, then –
(A) f (x) is discontinuous for $x = n^{1/3}$, where $n \in I$
(B) f (3/2) = 1
(C) f ' (x) = 0 for –1 < x < 1
(D) None of these

Q.16 If $f(x)=\frac{x-a}{\sqrt{x}-\sqrt{a}}$ is continuous at x = 1, then f (A) =
(A) $\sqrt{a}$ (B) $2\sqrt{a}$
(C) a (D) 2a

Q.17 Let f (x) be a continuous function for $1 \le x \le 3$. If f (x) takes rational values for all x and f (2) = 10, then f (1.5) is equal to
(A) 0 (B) 10
(C) not defined (D) any constant

Q.18 $\lim_{x\to\pi/2}\frac{\left(1-\tan\left(\frac{x}{2}\right)\right)(1-\sin x)}{\left(1+\tan\left(\frac{x}{2}\right)\right)(\pi-2x)^3}=?$
(A) 1/8 (B) 0
(C) 1/32 (D) ∞

Q.19 Let f(x) = $\lim_{x\to\infty}\sin^{2n}x$, then number of point(s) where f (x) is discontinuous is –
(A) 0 (B) 1
(C) 2 (D) infinitely many

Q.20 If $f(x) = (x + 1)^{\cot x}$ is continuous at $x = 0$, then $f(0)$ is equal to

(A) 0 (B) 1

(C) 1/e (D) e

For Q.21-Q.25 :

The answer to each question is a NUMERICAL VALUE.

Q.21 $\lim_{x\to 0^+} [1+[x]]^{2/x}$, where [.] is greatest integer function, is equal to –

Q.22 If $f(x) = \begin{cases} x\sin 1/x & , \ x \neq 0 \\ k & , \ x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k will be

Q.23 Suppose the function g is defined by

$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5 \end{cases}$, where k and m are constants. If g is derivable at $x = 3$ then $(k + m)$ equals –

Q.24 If the function f (x) defined by

$f(x) = \begin{cases} \dfrac{\log(1+3x) - \log(1-2x)}{x} & , \ x \neq 0 \\ a & , \ x = 0 \end{cases}$

is continuous at $x = 0$, then the value of a is –

Q.25 $\lim_{x\to 0} |x(x-1)|^{[\cos 2x]}$, where [x] represents greatest integer less than or equal to x.