Shiksha Classes Bhandara

Mathematics **Topic : Integration** **MM 100**

Q.1 The value of the definite integral

$I = \int_0^{\pi/2} e^x \left\{ \cos(\sin x)\cos^2\frac{x}{2} + \sin(\sin x)\sin^2\frac{x}{2} \right\} dx$, is –

(A) $\frac{1}{2}[e^{\pi/2}(\cos 1+\sin 1)-1]$ (B) $\frac{e^{\pi/2}}{2}(\cos 1+\sin 1)$

(C) $\frac{1}{2}(e^{\pi/2}\cos 1-1)$ (D) $\frac{e^{\pi/2}}{2}(\cos 1+\sin 1-1)$

Q.2 $I=\int \frac{1}{1-\cos^4 x}dx$ is equal to

(A) $2\sqrt{2}(\cot x+\sqrt{2}\tan^{-1}\sqrt{2}\cot x)$

(B) $-\frac{1}{2}\cot x-\frac{1}{2\sqrt{2}}\tan^{-1}\left(\cot x\sqrt{2}\right)+c$

(C) $2\sqrt{2}\{\cot x+\tan^{-1}(\cos x)\}$

(D) none of these

Q.3 Let f (x) be a polynomial of degree three satisfying f (0) = –1 and f (1) = 0. Also, 0 is a stationary point of f (x). If f (x) does not have an extremum at x = 0, then $\int \frac{f(x)}{x^3-1}dx$ is equal to –

(A) $\frac{x^2}{2}+C$ (B) x + C

(C) $\frac{x^3}{6}+C$ (D) None of these

Q.4 If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log[f(x)] + C$ then f (x) is equal to –

(A) $\frac{1}{a^2\sin^2 x-b^2\cos^2 x}$ (B) $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

(C) $\frac{1}{a^2\cos^2 x-b^2\sin^2 x}$ (D) None of these

Q.5 $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ =

(A) 0 (B) 1

(C) –π/2 (D) π/2

Q.6 Evaluate : $\int(\cos x-\sin x)(3+4\sin 2x)dx$

(A) $\left(\frac{\sin x-\cos x}{3}\right)(1+4\sin 2x)+c$

(B) $\left(\frac{\sin x-\cos x}{3}\right)(1-4\sin 2x)+c$

(C) $\left(\frac{\sin x+\cos x}{3}\right)(1+4\sin 2x)+c$

(D) None of these

Q.7 $\int_0^{\pi/2} x\left|\sin^2 x-\frac{1}{2}\right|dx$ is equal to

(A) π/2 (B) π/4

(C) π/8 (D) 3π/4

Q.8 $\int \frac{e^{x-1}}{(x^2-5x+4)}.2x\,dx = A\,F(x-1)+B(x-4)+c$,

where $F(x) = \int \frac{e^x}{x}dx$ then A and B ordered set is –

(A) $\left(-\frac{2}{3},\frac{8}{3}\right)$ (B) $\left(-\frac{2}{3},\frac{8e^3}{3}\right)$

(C) $\left(\frac{8}{3},\frac{2}{3}\right)$ (D) $\left(-\frac{2}{3},-\frac{8e^3}{3}\right)$

Q.9 Given $f'(x)=\frac{\cos x}{x}$, $f\left(\frac{\pi}{2}\right)=a$, $f\left(\frac{3\pi}{2}\right)=b$.

The value of the definite integral $\int_{\pi/2}^{3\pi/2} f(x)\,dx$ equals –

(A) $2-\frac{\pi}{2}(a-3b)$ (B) $2-\frac{\pi}{2}(a+3b)$

(C) $2+\frac{\pi}{2}(3a+b)$ (D) $2-\frac{\pi}{2}(3a-b)$

Q.10 Evaluate : $\int x^2\sin x\,dx$

(A) $-x^2\cos x + 2x\sin x - 2\cos x + c$

(B) $x^2\cos x + 2x\sin x - 2\cos x + c$

(C) $x^2\cos x + 2x\sin x + 2\cos x + c$

(D) None of these

Q.11 $\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$ is equal to

(A) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$ (B) $\left(\frac{x^2+1}{x^2}\right)^{n+6}(n+6)+c$

(C) $\left(\frac{x}{x^2+1}\right)^{n+6}(n+6)+c$ (D) none of these

Q.12 $\int \frac{1}{x(x^n+1)}dx$, (where n is a nonzero constant) is equal to

(A) $\frac{1}{n}\ell n\left|\frac{x^n}{x^n+1}\right|+C$ (B) $\frac{1}{n}\ell n\left|\frac{x^n+1}{x^n}\right|+C$

(C) $\ell n\left|\frac{x^n}{x^n+1}\right|+C$ (D) none of these

Q.13 Let $\int \frac{dx}{x^{2008}+x} = \frac{1}{p}\ln\left(\frac{x^q}{1+x^r}\right)+C$, where p, q, r ∈ N and need not be distinct, then the value of (p + q + r) equals

(A) 6024 (B) 6022

(C) 6021 (D) 6020

Q.14 If [x] stands for the greatest integer function, then $\int_1^3 [x]^x \, dx$

(A) Cannot be evaluated
(B) Has the value $1 + 4 (\log 2)^{-1}$
(C) Has the value $1 + 4 \log 2$
(D) Has the value $1 - 4 \log 2$

Q.15 $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = \pi \frac{\cos\alpha}{1+\sin^2\alpha}$

(A) for no value of α
(B) for exactly two values of α in $(0, \pi)$
(C) for at least one α in $\left(\frac{\pi}{2}, \pi\right)$
(D) for exactly one α in $\left(0, \frac{\pi}{2}\right)$

Q.16 $\int_{\pi/4}^{3\pi/4} \frac{\cos x}{1-\cos x} dx$ is equal to-

(A) $2 - \frac{\pi}{2}$ (B) $2 + \frac{\pi}{2}$
(C) $\frac{1}{2} - 2$ (D) $\frac{1}{2} + 2$

Q.17 If $f(t) = \begin{cases} at-1, & t<1 \\ t^2+b, & t\geq 1 \end{cases}$ then possible set of value of (a, b) so that $\int_0^x f(x)\,dx$ is differentiable for all is –

(A) (5, 1) (B) (1, 3)
(C) (4, 2) (D) None of these

Q.18 $\lim_{n\to\infty} \frac{1}{n}(e^{1/n} + e^{2/n} + e^{3/n} + ... + e^{n/n})$ is equal to

(A) e (B) e – 1
(C) 1 – e (D) None of these

Q.19 $\int_0^4 \{\sin\{x\}\} dx = \lambda$, where { x } represents fractional part of x. Then $\lambda/4$ is –

(A) – cos 1 (B) 1 – cos 1
(C) 1 – sin 1 (D) None of these

Q.20 Let f(x) = x – [x], for every real number x, where [x] is integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is

(A) 1 (B) 2
(C) 0 (D) 1/2

For Q.21-Q.25 :
The answer to each question is a NUMERICAL VALUE.

Q.21 Suppose for every integer n, $\int_n^{n+1} f(x)\,dx = n^2$.

The value of $\int_{-2}^{4} f(x)\,dx$ is –

Q.22 If $A = \int_1^{\sin\theta} \frac{t\,dt}{1+t^2}$ and $B = \int_1^{\cos ec\theta} \frac{dt}{t(1+t^2)}$, then the value of $\begin{vmatrix} A & A^2 & B \\ e^A e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}$ is –

Q.23 $\int_0^2 \frac{2x^3 - 6x^2 + 9x - 5}{x^2 - 2x + 5} dx$ is equal to

Q.24 Evaluate $\int_{-3/2}^{10} \{2x\} dx = \frac{23}{X}$. Find the value of X.
where {.} denotes the fractional part of x.

Q.25 Evaluate $\int_{-1}^{1} f(x)dx$, where $f(x) = \begin{cases} 1-2x, & x\leq 0 \\ 1+2x, & x\geq 0 \end{cases}$.