SHIKSHA CLASSES, BHANDARA

TEST-7

CHEMISTRY , PHYSICS, MATHEMATICS

Time : - 3 Hours **Max. Marks:- 300** **Date : .....**

INSTRUCTIONS :

1. The test is of 3 hours duration.
2. The Test Booklet consists of 90 questions. The maximum marks are **300.**
3. There are three parts in the question paper A, B, C consisting of Chemistry, Physics and Mathematics having **30questions** in each part of equal weightage. **20 questions** will be **MCQs** and **10 questions** (**ATTEMPT ANY FIVE QUESTIONS OUT OF 10**)will have answer to be filled as **numerical value**.
   Marking Scheme for MCQs
   Correct Answer Four mark (+4), Incorrect Answer Minus one mark (–1), Unanswered No mark (0)
   Marking Scheme for questions for which answer is a **Numerical value**
   Correct Answer Four mark (+4), Incorrect Answer No mark (0), Unanswered No mark (0)
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly.

SYLLABUS

CHEMISTRY: ORGANIC COMPOUNDS CONTAINING HALOGENS, ORGANIC COMPOUNDS CONTAINING OXYGEN.

PHYSICS : OPTICS, DUAL NATURE OF MATTER AND RADIATION, ATOMS AND NUCLEI

MATHEMATICS INTEGRAL CALCULUS, AREA UNDER CURVE, DIFFERENTIAL EQUATIONS

Work alone is Noble.

If a window of Opportunity appears, don't pull down the shade.

Name : .....

Address : .....

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Phone/Mobile No. .....

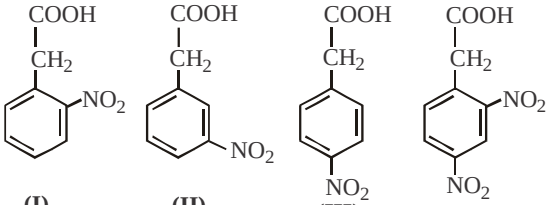
Roll No. .....

PART A – CHEMISTRY

SECTION - 1 (Q.1 - Q.20)

Each question has FOUR options (1), (2), (3) and (4). ONLY ONE of these four options is correct.

Q.1 The correct order for the rate of decarboxylation of following compounds with sodalime will be :-

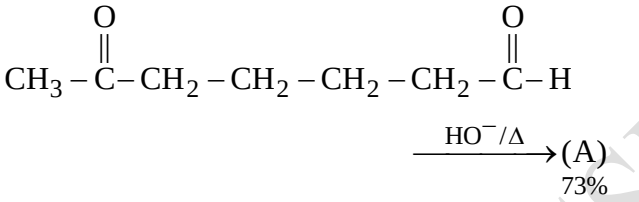


(I) **(II)** **(III)** **(IV)**

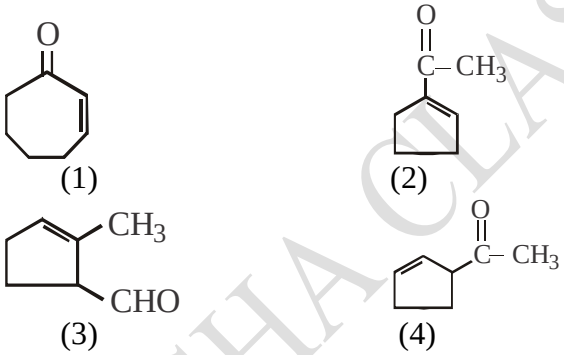
(1) IV > I > III > II (2) III > IV > I > II

(3) II > III > IV > I (4) I > II > III > IV

Q.2



Product A is :-



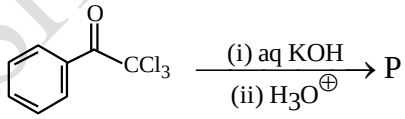
(1) (2) (3) (4)

Q.3 Which of the following alcohol has the highest boiling point?

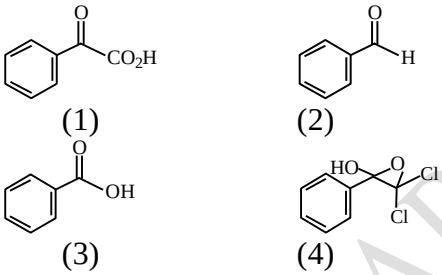
(1) Butan-2-ol (2) 2-Methylpropan-2-ol

(3) Propan-2-ol (4) Butan-1-ol

Q.4 One of the products of the following reactions P.

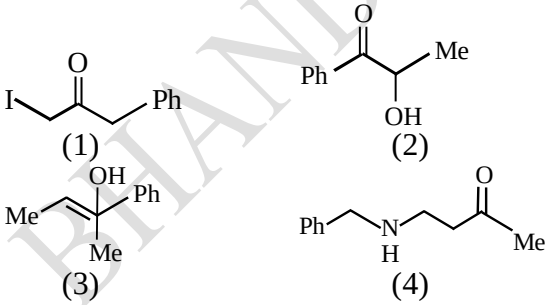


Structure of P is



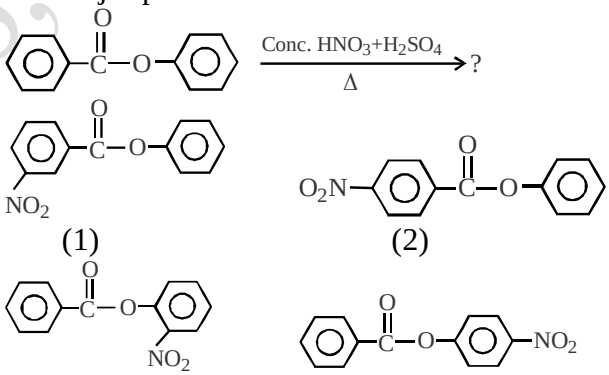
(1) (2) (3) (4)

Q.5 Haloform reaction with I_2 and KOH will be responded by



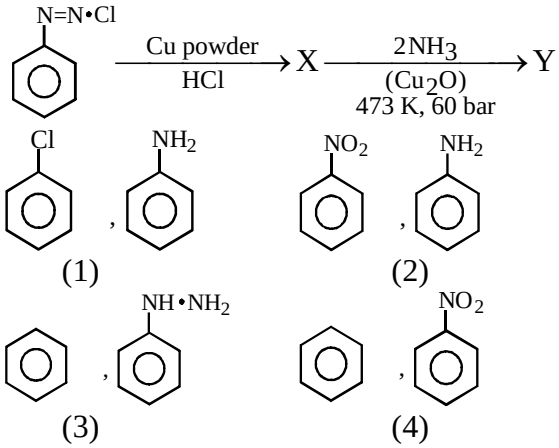
(1) (2) (3) (4)

Q.6 The major product in reaction is :



(1) (2) (3) (4)

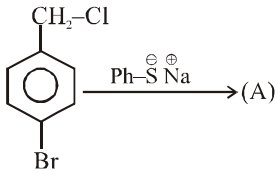
Q.7 Identify X and Y in following reaction.



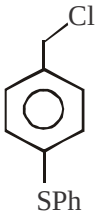
(1) (2) (3) (4)

SPACE FOR ROUGH WORK

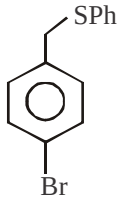
Q.8



Structure of product (A) is :-



(1)



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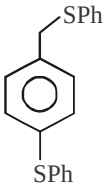
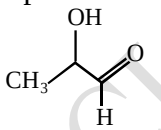


(3)

(4) None

Q.9 Oxidation of allyl alcohol with a peracid gives a compound of molecular formula $C_3H_6O_2$, which contains an asymmetric carbon atom. The structure of the compound is

(1)



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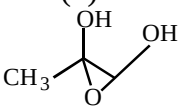

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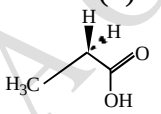


(4)

Q.10 In which of the following S_N1 reaction racemic mixture will form as major product ?

(1) $CH_3-\underset{\underset{H}{|}}{\overset{H}{\overset{|}{C}}}-CH_2-OH \xrightarrow{HI}$ Product

(2) $CH_3-\underset{\underset{CH_3}{|}}{\overset{CH_3}{\overset{|}{C}}}-CH_2-Br \xrightarrow[\text{(Polar solvent)}]{\text{NaOH (aq)}}$ Product

(3) $Ph-\underset{\underset{CH_3}{|}}{\overset{CH_3}{\overset{|}{C}}}-CH_2-OH \xrightarrow{HCl/ZnCl_2}$ Product

(4) $Ph-\underset{\underset{H}{|}}{\overset{Ph}{\overset{|}{C}}}-CH_2-Cl \xrightarrow{CH_3OH}$ Product

Q.11 $P \xrightarrow[KOH]{KMnO_4} Q \xrightarrow[\Delta]{\text{Sodalime}} R \xrightarrow[\text{Anhyd. } AlCl_3]{CH_2Cl} S$

If P and S are toluene, Q & R are _____ and _____ respectively.

(1) Benzaldehyde, Benzoic acid
(2) Benzaldehyde, Sodium benzoate
(3) Benzoic acid, Benzene
(4) Benzene, Benzoic acid

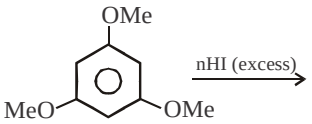
Q.12 In the following sequence of reactions

$$CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow[\text{Ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

then compound 'D' is -

(1) butanal (2) n–butyl alcohol
(3) n–propyl alcohol (4) propanal

Q.13

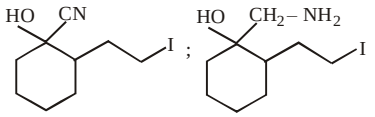


Value of n in above reaction is :

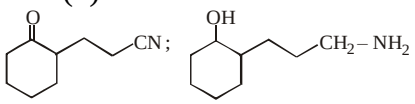
(1) 3 (2) 4
(3) 5 (4) 6

SPACE FOR ROUGH WORK

Q.14 The major products A and B for the following reactions are, respectively:



(1)



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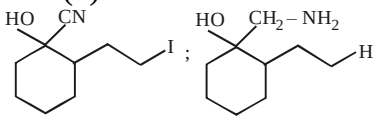

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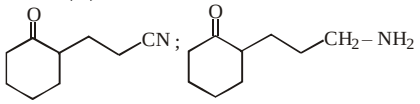
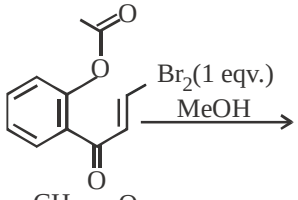
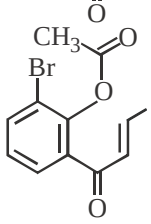


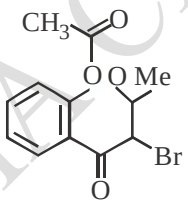
(4)

Q.15 The major product obtained in the following conversion is :





(1)



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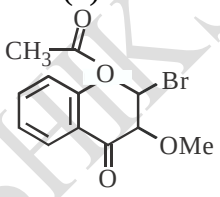

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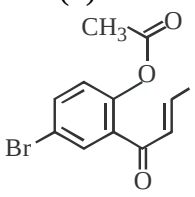
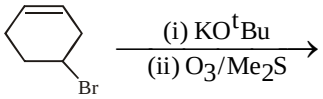
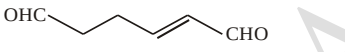
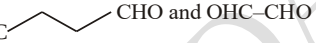


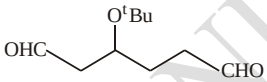
(4)

Q.16 The major product(s) obtained in the following reaction is/are :



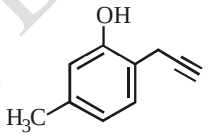
(1) 

(2) 

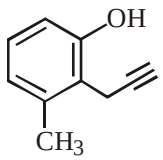
(3) 

(4) 

Q.17 What will be the major product when m-cresol is reacted with propargyl bromide ($HC\equiv C-CH_2Br$) in present of K_2CO_3 in acetone



(1)



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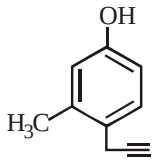

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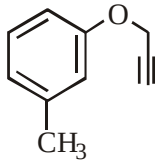
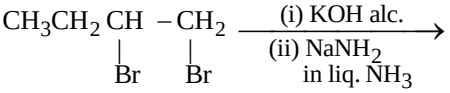
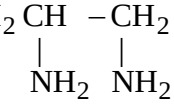


(4)

Q.18 The major product of the following reaction is:



(1) $CH_3CH_2C \equiv CH$

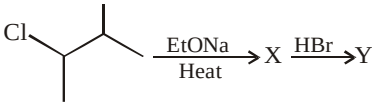
(2) 

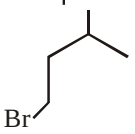
(3) $CH_3CH = C = CH_2$

(4) $CH_3CH = CHCH_2NH_2$

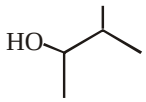
SPACE FOR ROUGH WORK

Q.19 The major product 'Y' in the following reaction is:





(1)



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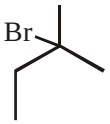

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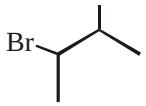
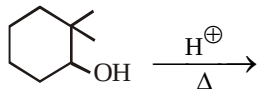
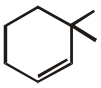


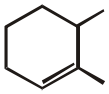
(4)

Q.20 

Major product is :



(1)



(2)

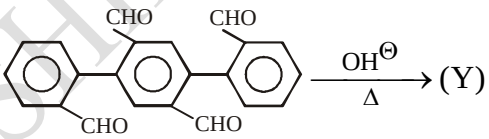
(3)

(4)

SECTION - 2 (Q.21 - Q.30)

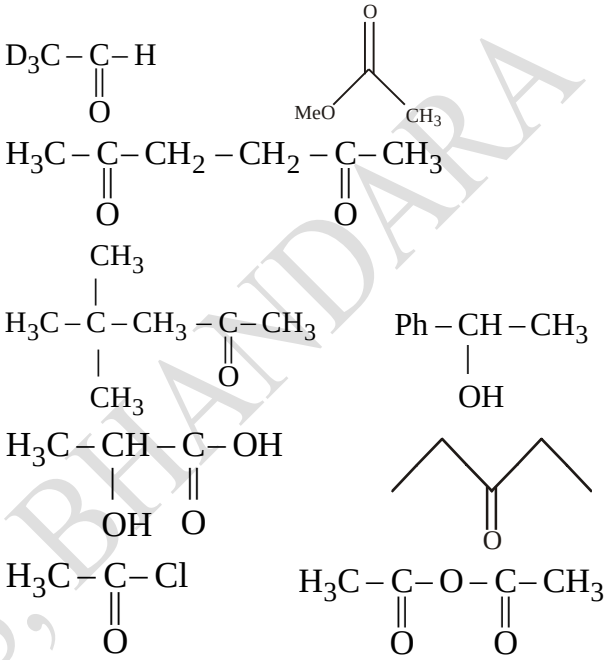
This section contains TEN (10) questions. ATTEMPT ANY FIVE (05) QUESTIONS. The answer to each question is NUMERICAL VALUE. If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Q.21 How many total isomers of molecular formula $C_5H_{11}Cl$ will form 2-Methylbutan-2-ol on hydrolysis ?

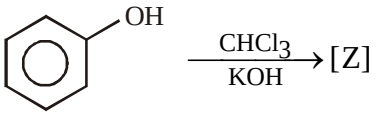
Q.22 

[No. of isomeric product obtained].

Q.23 Number of compounds which can give Haloform reaction, are.....



Q.24 Number of organic product(s) obtained in significant yield



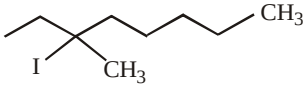
Q.25 Identify the number of (OH) group in an unknown alcohol of molecular mass 90 which on reaction with acetyl chloride given ester of molecular mass 174 ?

Q.26 Two stereoisomers of 3, 4-Dimethylcyclopentane-1, 1-dicarboxylic acid undergo decarboxylation on heating. How many products are formed ?

Q.27 Number of moles of HIO_4 consumed during complete oxidation of one mole of Erythrose is

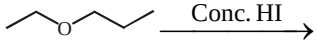
Q.28 Number of γ hydrogen atoms present in benzaldehyde is

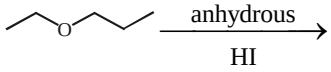
Q.29 How many distinct alkene products are possible when the alkyl iodide below undergoes E2 elimination ?



SPACE FOR ROUGH WORK

Q.30 Sum of molecular mass of iodides produced in following reaction is 450 + A then find the value of A

(a) 

(b) 

(c) $Ph-O-Me \xrightarrow[excess]{HI}$

PART B – PHYSICS

SECTION - 1 (Q.31 - Q.50)

Each question has FOUR options (1), (2), (3) and (4). ONLY ONE of these four options is correct.

Q.31 A glass concave lens is placed in a liquid in which it behaves like a convergent lens. If the refractive indices of glass and liquid with respect to air are ${}^a\mu_g$ and ${}^a\mu_\ell$ respectively, then:-

(1) ${}^a\mu_g = 5\,{}^a\mu_\ell$ (2) ${}^a\mu_g > {}^a\mu_\ell$

(3) ${}^a\mu_g < {}^a\mu_\ell$ (4) ${}^a\mu_g = 2\,{}^a\mu_\ell$

Q.32 Photons of energy 2eV and 2.5eV successively illuminate a metal whose work function is 0.5eV. The ratio of maximum speed of emitted electron is

(1) $\sqrt{3}:2$ (2) 2 : 1

(3) 1 : 2 (4) $2:\sqrt{3}$

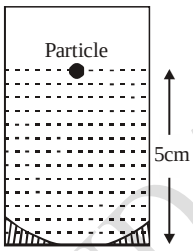
Q.33 In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1) $(\sqrt{3}+1)^4 : 16$ (2) 9 : 1

(3) 4 : 1 (4) 25 : 9

Q.34 A concave mirror has radius of curvature of 40cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to :

(Refractive index of water = 1.33)



(1) 8.8 cm (2) 11.7 cm

(3) 6.7 cm (4) 13.4 cm

Q.35 A point object is placed on the axis of a thin convex lens of focal length 0.05 m at a distance of 0.2 m from the lens and its image is formed on the axis. If the object is now made to oscillate along the axis with a small amplitude of A cm, then what is the amplitude of oscillation of the image? [you may assume, $\frac{1}{1+x} \approx 1-x$, where $x << 1$]

(1) $\frac{4A}{9}\times10^{-2}$m (2) $\frac{5A}{9}\times10^{-2}$m

(3) $\frac{A}{3}\times10^{-2}$m (4) $\frac{A}{9}\times10^{-2}$m

Q.36 In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ?

(Given : $h = 6.63 \times 10^{-34}$ Js ; $c = 3 \times 10^8$ ms^{-1})

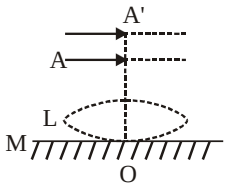
(1) 9.4 nm (2) 12.3 nm

(3) 10.8 nm (4) 11.4 nm

Q.37 A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, The pin has to be moved to A', such that OA' = 27 cm,

SPACE FOR ROUGH WORK

to get its inverted real image at A' itself. The value of μ_1 will be :-

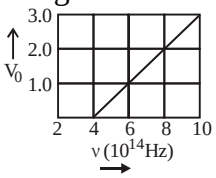


(1) $\sqrt{2}$ (2) 4/3

(3) $\sqrt{3}$ (4) 3/2

Q.38 The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be :

(Given : Planck's constant (h) = 6.63×10^{-34} Js, electron charge e = 1.6×10^{-19} C)



(1) 1.95 eV (2) 1.82 eV

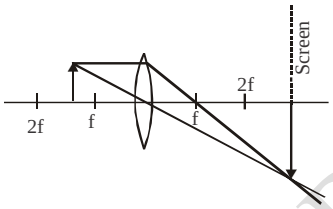
(3) 1.66 eV (4) 2.12 eV

Q.39 A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :

(1) 20 cm (2) 10 cm

(3) 25 cm (4) 30 cm

Q.40 Formation of real image using a biconvex lens is shown below :



If the whole set up is immersed in water without disturbing the object and the screen position, what will one observe on the screen ?

(1) Image disappears (2) No change

(3) Erect real image (4) Magnified image

Q.41 In Young's experiment fourth bright fringe produced by light of 5000Å superposes on the fifth bright fringe of an unknown wavelength. The unknown wavelength is ___Å.

(1) 4000 (2) 6000

(3) 5000 (4) 8000

Q.42 Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm (2) 642.7 nm

(3) 488.9 nm (4) 388.9 nm

Q.43 In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1) $\frac{\lambda}{2(\mu-1)}$ (2) $\frac{\lambda}{2\mu-1}$

(3) $\frac{2\lambda}{\mu-1}$ (4) $\frac{\lambda}{\mu-1}$

Q.44 Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 1 : 8

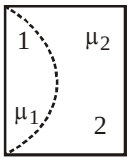
(3) 8 : 1 (4) 3 : 8

SPACE FOR ROUGH WORK

Q.45 A 2 mW laser operates at a wavelength of 500nm. The number of photons that will be emitted per second is :
[Given Planck's constant h = 6.6×10^{-34} Js,
Speed of light c = 3.0×10^{8} m/s]
(1) 2×10^{16} (2) 1.5×10^{16}
(3) 5×10^{15} (4) 1×10^{16}

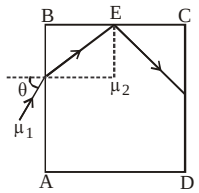
Q.46 If the tube length of astronomical telescope is 96cm and magnifying power is 15 for normal setting, then the focal length of the objective is _____ cm.
(1) 100 (2) 90
(3) 105 (4) 92

Q.47 One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\frac{R}{2-(\mu_1-\mu_2)}$ (2) $\frac{2R}{\mu_1-\mu_2}$
(3) $\frac{R}{2(\mu_1-\mu_2)}$ (4) $\frac{R}{\mu_1-\mu_2}$

Q.48 A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index μ_1 ($\mu_1 < \mu_2$). A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC. The θ must satisfy :



(1) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (2) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$
(3) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (4) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

Q.49 A proton and an electron initially at rest are accelerated by the same potential difference. Assuming that a proton is 2000 times heavier than an electron, what will be the relation between the de Broglie wavelength of the proton (λ_p) and that of electron (λ_e) ?
(1) $\lambda_p = 2000\,\lambda_e$ (2) $\lambda_p = \frac{\lambda_e}{2000}$
(3) $\lambda_p = 20\sqrt{5}\,\lambda_e$ (4) $\lambda_p = \frac{\lambda_e}{20\sqrt{5}}$

Q.50 If the kinetic energy of the electron in the hydrogen atoms is $\frac{e^2}{8\pi\varepsilon_0 r}$, then its potential energy is –
(1) $\frac{e^2}{4\pi\varepsilon_0 r}$ (2) $-\frac{e^2}{4\pi\varepsilon_0 r}$ (3) $\frac{e^2}{8\pi\varepsilon_0 r}$ (4) $-\frac{e^2}{8\pi\varepsilon_0 r}$

SECTION - 2 (Q.51 - Q.60)

This section contains TEN (10) questions. ATTEMPT ANY FIVE (05) QUESTIONS. The answer to each question is NUMERICAL VALUE. If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Q.51 A small object is embedded in a glass sphere (μ = 1.5) of radius 5.0 cm at a distance 1.5 cm left to the centre. The image of the object as seen by an observer standing to the left of the sphere is at (X cm) left to the centre. Find the value of X.

SPACE FOR ROUGH WORK

Q.52 Light of wavelength 6000Å is incident on a slit of width 0.30 mm. The screen is placed 2m from slit. The position of the first minima is $(X) \times 10^{-3}$ m. Find the value of X.

Q.53 In an experiment on photoelectric effect, the wavelength of the incident radiation is λ. The wavelength of the incident radiation is reduced to 1/3 of the initial value and the maximum kinetic energy of the photoelectron is observed to be n times the previous value. The threshold wavelength for the metal plate is $\left(\frac{n-1}{n-X}\right)\lambda$. Find the value of X.

Q.54 In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light is

Q.55 A radioactive sample contains two radio nucleoids A and B having decay constant λ hr^{-1} & 2λ hr^{-1}. Initially 25% of total decay comes from A. How long (in hr) will it take before 75% of total decay comes from A. [Take $\lambda = \ln 3$]

Q.56 An equiconvex lens has a power of 5 diopter. If it is made of glass of refractive index 1.5 then the radius of the curvature of each surface is $(5 \times X)$ cm. Find the value of X.

Q.57 A thin prism of angle A = 6° produces a deviation δ = 3 °. If the refractive index of the material of prism is x/2. Find the value of x.

Q.58 In Bohr's model, If 'B' and 'a' denote the magnetic field at centre and centripetal acceleration, where electron is in n^{th} orbit, then it is found that $\frac{a}{B} \propto \frac{n^{\alpha}}{z^{\beta}}$, where z = atomic no., then $\alpha + \beta$ is :

Q.59 The only source of energy in a particular star is the fusion reaction given by :

$3_2He^4 \rightarrow {}_6C^{12}$ + energy

Masses of $_2He^4$ and $_6C^{12}$ are given

m ($_2He^4$) = 4.0025u m ($_6C^{12}$) = 12.0000 u

Speed of light in vacuum is 3×10^8 m/s. Power output of star is 4.5×10^{27} watt. The rate at which the star burns helium is $n \times 10^{13}$ kg/s. Here n is an integer. Find n.

Q.60 About 185 MeV of usable energy is released in the neutron-induced fissioning of a $^{235}_{92}U$ nucleus. $^{235}_{92}U$ in a reactor is continuously generating 100MW of power. If the number of days that it will take for 1 kg of the uranium to be used up is $n \times 10^5$, find nearest integral value of n.

PART C – MATHEMATICS

SECTION - 1 (Q.61 - Q.80)

Each question has FOUR options (1), (2), (3) and (4). ONLY ONE of these four options is correct.

Q.61 If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of integral $\int_{-\pi/4}^{\pi/4} g(f(x))\,dx$ is

(1) $\log_e 3$ (2) $\log_e 2$

(3) $\log_e e$ (4) $\log_e 1$

Q.62 The area (in sq. units) of the region $A = \{(x,y) : \frac{y^2}{2} \le x \le y + 4\}$ is

(1) 53/3 (2) 18

(3) 30 (4) 16

Q.63 The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0$ $(x \neq 0)$ is $a\sqrt{2} + b$, : (where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 + 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 - 2x^2 + cx^3 = 0$

Q.64 If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le l, x \ge 0, y \ge 0\}$ is then a – b is equal to :

(1) 8/3 (2) 10/3

(3) 6 (4) –2/3

SPACE FOR ROUGH WORK

Q.65 Let y = y (x) be the solution of the differential equation, $\frac{dy}{dx}$ + y tan x = 2x + x^2 tan x, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y (0) = 1. Then :

(1) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) - y\left(\frac{-\pi}{4}\right) = \sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right) + y\left(\frac{-\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Q.66 The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with y (1) = 1, is

(1) $y = \frac{x^3}{5} + \frac{1}{5x^2}$ (2) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$

(3) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$ (4) $y = \frac{x^2}{4} + \frac{3}{4x^2}$

Q.67 The area (in sq. units) of the region
A = {(x, y) ∈ R × R | 0 ≤ x ≤ 3, 0 ≤ y ≤ 4, $y \le x^2 + 3x$} is :

(1) 53 / 6 (2) 59 / 6
(3) 8 (4) 26 / 3

Q.68 The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to (C is a constant of integration)

(1) $3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$

(3) $-3\cot^{-1/3}x + C$ (4) $-3\tan^{-1/3}x + C$

Q.69 If y = y (x) is the solution of the differential equation $\frac{dy}{dx}$ = (tan x – y) $\sec^2$ x, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y (0) = 0, then y $(-\pi/4)$ is equal to :

(1) $2 + \frac{1}{e}$ (2) $\frac{1}{2} - e$

(3) e – 2 (4) $\frac{1}{2} + e$

Q.70 $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx$ is equal to :
(where c is a constant of integration)
(1) 2x + sin x + 2sin 2x + c
(2) x + 2sin x + 2sin 2x + c
(3) x + 2sin x + sin 2x + c
(4) 2x + sin x + sin 2x + c

Q.71 If $\int \sin^{13} x \cos^3 x\, dx = A\sin^{14} x + B\sin^{16} x + C$, then A + B = ……..

(1) $\frac{1}{110}$ (2) $\frac{17}{112}$

(3) $\frac{15}{112}$ (4) $\frac{1}{112}$

Q.72 A curve passes through the point (1, π/4) & its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation -

(1) $y = x\tan^{-1}\left(\ell n\frac{e}{x}\right)$ (2) $y = x\tan^{-1}(\ell n + 2)$

(3) $y = \frac{1}{x}\tan^{-1}\left(\ell n\frac{e}{x}\right)$ (4) none

Q.73 $\int\left(\frac{\cos^8 x - \sin^8 x}{1 - 2\sin^2 x\cos^2 x}\right) dx$ equals -

(1) $-\frac{\sin 2x}{2} + c$ (2) $\frac{\sin 2x}{2} + c$

(3) $\frac{\cos 2x}{2} + c$ (4) $-\frac{\cos 2x}{2} + c$

Q.74 The solution of $y^5 x + y - x\frac{dy}{dx} = 0$ is -

(1) $x^4/4 + 1/5\,(x/y)^5 = C$
(2) $x^5/5 + (1/4)\,(x/y)^4 = C$
(3) $(x/y)^5 + x^4/4 = C$ (4) $(xy)^4 + x^5/5 = C$

SPACE FOR ROUGH WORK

Q.75 The value of the definite integral

$\int_1^e \left((x+1)e^x.\ell nx\right)dx$ is -

(1) e (2) e^{e+1}

(3) $e^e(e-1)$ (4) $e^e(e-1)+e$

Q.76 The area bounded by the curve $y = xe^{-x}$; xy= 0 and x = c, where c is the x-coordinate of the curve's inflection point, is -

(1) $1-3e^{-2}$ (2) $1-2e^{-2}$

(3) $1-e^{-2}$ (4) 1

Q.77 Let a, b, c be non-zero real numbers such that

$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx$

$= \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\,dx,$

then the quadratic equation $ax^2 + bx + c = 0$ has

(1) no root in (0,2)

(2) atleast one root in (0,2)

(3) a double root in (0,2)

(4) none

Q.78 Let $I = \int_a^b (x^4 - 2x^2)\,dx$. If I is minimum then the ordered pair (a, b) is :

(1) $(-\sqrt{2}, 0)$ (2) $(-\sqrt{2}, \sqrt{2})$

(3) $(0, \sqrt{2})$ (4) $(\sqrt{2}, -\sqrt{2})$

Q.79 The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

Q.80 The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)\,dx + 2xy\,dy = 0$ which passes through (1, 1) is :

(1) A circle with centre on the y-axis.

(2) A circle with centre on the x-axis.

(3) An ellipse with major axis along the y-axis.

(4) A hyperbola with transverse axis along the x-axis.

SECTION - 2 (Q.81 - Q.90)

The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.

Q.81 If $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}dx$, $x \neq 0$ then $\lim_{x\to 0} f'(x)$ is equal to-

Q.82 Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$ is -

Q.83 If $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$ then $y\left(-\frac{\pi}{4}\right) = \frac{1}{3} + e^A$. Find the value of A.

Q.84 The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is A/8. Find the value of A.

Q.85 The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$

(where [x] denotes the greatest integer less than or equal to x) is :

SPACE FOR ROUGH WORK

Q.86 If the differential equation of a curve, passing through $(0, -\pi/4)$ and $(t, 0)$ is

$$\cos y\left(\frac{dy}{dx}+e^{-x}\right)+\sin y\left(e^{-x}-\frac{dy}{dx}\right)=e^{e^{-x}}$$

then find the value of $t\cdot e^{e^{-t}}$.

Q.87 Let $f : R \to R$ be a twice differentiable function satisfying $f(2) = -1$, $f'(2) = 4$ and $\int_2^3 (3-x)\, f''(x)\, dx = 7$, then the value of $(f(3) - 3)$ is equal to

Q.88 If $\int \frac{dx}{\sqrt[3]{x^2}(1+\sqrt[3]{x^2})} = a\tan^{-1}(x^b)+c$ then $a + 3b$ is

Q.89 Let $A = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{2nr-r^2}}$ then $\frac{2A}{\pi}$ is

Q.90 If $\int_{\sin x}^{1} t^2 f(t)\, dt = 1-\sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is –

SPACE FOR ROUGH WORK