Shiksha Classes Bhandara

Mathematics **Topic : Differentiation** **MM 100**

Q.1 If $f(x) = \frac{x-4}{2\sqrt{x}}$ then $f'(0)$ is

(A) 0 (B) 1
(C) Does not exist (D) None of these

Q.2 If $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$ then find $f'(\pi/4)$.

(A) 1 (B) 2
(C) $\sqrt{2}$ (D) 0

Q.3 If g is the inverse of f and $f'(x) = \frac{1}{1+x^3}$, then find $g'(x)$.

(A) $1 + [g(x)]^{-1}$ (B) $1 + [g(x)]^2$
(C) $1 + [g(x)]^3$ (D) $1 - [g(x)]^3$

Q.4 If $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$ and $\frac{dy}{dx} = ax + b$ then the value of $a + b$ is equal to

(A) $\cot\frac{5\pi}{8}$ (B) $\cot\frac{5\pi}{12}$
(C) $\tan\frac{5\pi}{12}$ (D) $\tan\frac{5\pi}{8}$

Q.5 If $x^y + y^x = a^b$ find $\frac{dy}{dx}$.

(A) $-\frac{y\,x^y + y^x \log y}{x^y \log x + xy^x}$ (B) $-\frac{y\,x^{y+1} - y^x \log y}{x^y \log x + xy^{x+1}}$

(C) $-\frac{y\,x^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}$ (D) $-\frac{y\,x^y + y^x \log x}{x^y \log x + xy^{x+1}}$

Q.6 Let $e^{f(x)} = \ln x$. If $g(x)$ is the inverse function of $f(x)$ then $g'(x)$ equals to :

(A) e^x (B) $e^x + x$
(C) $e^{(x + e^x)}$ (D) $e^{(x + \ln x)}$

Q.7 Find dy/dx for the function of x defined implicitly by the equation $x^2 + y^2 = 4$.

(A) $-x/y$ (B) x/y
(C) y/x (D) x/y

Q.8 Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the product ab is equal to

(A) 25 (B) 9
(C) – 15 (D) – 9

Q.9 If $\phi(x) = \log_5 \log_3 x$, then $\phi'(e)$ is equal to –

(A) e log 5 (B) – e log 5
(C) $\frac{1}{e\log 5}$ (D) None of these

Q.10 If $x^2 + y^2 = R^2$ $(R > 0)$ then $k = \frac{y''}{\sqrt{(1 + y'^2)^3}}$ where k in terms of R alone is equal to

(A) $-1/R^2$ (B) $-1/R$
(C) $2/R$ (D) $-2/R^2$

Q.11 Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is –

(A) 1 (B) e^{-2x}
(C) $2e^{-2x}$ (D) $-2e^{-2x}$

Q.12 If $F(x) = f(x)\,g(x)\,h(x)$ where f, g, h are differentiable and always give positive values of all real values of x. If $F'(x_0) = 13F(x_0)$ and $f'(x_0) = 9f(x_0)$ and $g'(x_0) = 4g(x_0)$, then the value of $\frac{h'(x_0)}{h(x_0)}$, where $x_0 \in R$ is –

(A) 17 (B) 13
(C) 9 (D) None of these

Q.13 If $f(x) = |\cos x|$, then $f'(3\pi/4)$ is equal to –

(A) $-1/\sqrt{2}$ (B) $1/\sqrt{2}$
(C) 1 (D) None of these

Q.14 A non zero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$. The leading coefficient of $f(x)$ is

(A) 1/6 (B) 1/9
(C) 1/12 (D) 1/18

Q.15 If $y = x^3 \log x$, then value of y_4 is

(A) 6x (B) – 6x
(C) 6/x (D) – 6/x

Q.16 If $y = \sin x$ then the value of $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

(where suffixes of y shows the order of derivative) is –

(A) independent of x but dependent on m
(B) dependent of x but independent of m
(C) dependent on both m and x
(D) independent of m and x

Q.17 $\frac{d}{dx}\left[\sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right]$ equals

(A) –1 (B) 1/2
(C) –1/2 (D) 1

Q.18 Find the differential coefficient of $\tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$ w.r.t 'x'.

(A) 1 (B) 2
(C) 3 (D) None of these

Q.19 If $f(t) = \frac{1-t}{1+t}$, then $f'(1/t)$ is equal to

(A) $\frac{1}{(1+t)^2}$ (B) $\frac{1}{(t-1)^2}$

(C) $\frac{-2t^2}{(t+1)^2}$ (D) $\frac{2}{(t-1)^2}$

Q.20 If $y = \sin m\theta$ and $x = \sin\theta$, find $(1 - x^2)y_2 - xy_1 + m^2y = ?$

(A) 2 (B) 0
(C) 1 (D) None of these

For Q.21-Q.25 :

The answer to each question is a NUMERICAL VALUE.

Q.21 Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$.

Q.22 Let $x\sqrt{1+y}-y\sqrt{1+x}=0$, where $-1< x < 0$, for this relation find the sum of squares of all values of $\frac{dy}{dx}$ at $x=-\frac{1}{2}$.

Q.23 If $y = \sin^{-1} x$, find $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ = ?

Q.24 If $y = \cos^{-1}\left(\frac{5\cos x-12\sin x}{13}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to –

Q.25 If $y = e^{-x}\cos x$ and $y_4 + ky = 0$, where $y_4 = \frac{d^4y}{dx^4}$, then k =