SHIKSHA CLASSES

BOARD ANSWER PAPER

Subject : Maths- I **Topic: 6. Line and Plane** **Total Marks : 20**

Class : XII **7. Linear Programming**

Section - A (3 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) The vector equation of a line which passes through the point with position vector $4\hat{i}-\hat{j}+2\hat{k}$ and is in the direction of $-2\hat{i}+\hat{j}+\hat{k}$ is.

Ans.: (b) $\bar{r}=(4\hat{i}-\hat{j}+2\hat{k})+\lambda(-2\hat{i}+\hat{j}+\hat{k})$

$\bar{a}=4\hat{i}-\hat{j}+2\hat{k}$ and $\bar{b}=-2\hat{i}+\hat{j}+\hat{k}$

The equation of line in vector form is

$\bar{r}=\bar{a}+\lambda\bar{b}$

$\bar{r}=(4\hat{i}-\hat{j}+2\hat{k})+\lambda(-2\hat{i}+\hat{j}+\hat{k})$

2) The two planes $3x-6y-2z=7$ and $2x+y-kz=5$ are prpendicular to each other then value of k is equal to

Ans: (d) 0

The d.r.s. of the two planes are 3, –6, –2, and 2, 1, k respectively.

$\therefore$ The planes are perpendicular.

$\therefore$ Its normals are perpendicular.

$\therefore\ 3(2)+(-6)(1)+(-2)(k)=0$

$\therefore\ 6-6-2k=0$

$\therefore\ 0=2k$

$\therefore\ k=0$.

3) The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is

Ans:a) $\frac{1}{\sqrt{6}}$

First line with d.r.'s 2, 3, 4 passes through A (1, 2, 3).

Second line with d.r.'s 3, 4, 5 passes through B(2, 4, 5).

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1$$

$$\sqrt{(b_1c_2-b_2c_1)^2+(a_2c_1-a_1c_2)^2+(a_1b_2-a_2b_1)^2}$$

$$= \sqrt{1+4+1}=\sqrt{6}$$

$\therefore$ Shortest distance $=\left|\frac{1}{\sqrt{6}}\right|=\frac{1}{\sqrt{6}}$ units.

Section - B (6 Marks)

4) Find the vector equation of the line passing through the point (4,2,7) and parallel to the vector $3\hat{i}-5\hat{j}$

Ans. : $\bar{a}=4\hat{i}+2\hat{j}+7\hat{k}$, $\bar{b}=3\hat{i}-5\hat{j}$

The equation of line in vector form is

$\bar{r}=\bar{a}+\lambda\bar{b}$

$\bar{r}=\left(4\hat{i}+2\hat{j}+7\hat{k}\right)+\lambda\left(3\hat{i}-5\hat{j}\right)$

5) The cartesian equation of the line is 6x – 2 = 3y + 1 = 2z – 2 find its direction ratio.

Ans. : $6x-2=3y+1=2z-2$

$\therefore 6\left(x-\frac{1}{3}\right)=3\left(y+\frac{1}{3}\right)=2(z-1)$

$$\therefore \frac{x-\frac{1}{3}}{\frac{1}{6}} = \frac{y+\frac{1}{3}}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$$ dr's are

$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ i.e. 1 , 2 , 3

6) Find the direction ratios of a line passing through the points (–2,1,–8) and (4,3,–5)

Ans: Let A (–2,1,–8) and B(4,3,–5)

dr's are 4 + 2 , 3 – 1, –5 + 8

i.e. 6 , 2 , 3

OR

Find λ if the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-\lambda}$ and $\frac{x-1}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar.

Ans: The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-\lambda}$ and $\frac{x-1}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -\lambda \\ \lambda & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1+2\lambda) + 1(1+\lambda^2) - 1(2-\lambda) = 0$$

$$\Rightarrow 2\lambda + 1 + 1 + \lambda^2 - 2 + \lambda = 0$$

$$\Rightarrow \lambda^2 + 3\lambda = 0 \Rightarrow \lambda(\lambda+3) = 0 \Rightarrow \lambda = 0 \; or \; -3$$

Section - C (9 Marks)

7) Find the cartesian equation of the plane $\bar{r}.(\hat{i}-\hat{j}) + \lambda(\hat{i}+\hat{j}+\hat{k}) + \mu(\hat{i}-2\hat{j}+3\hat{k})$.

Ans.: Given plane is perpendicular to vector $\bar{n}$, where

$$\bar{n} = \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 5\hat{i} - 2\hat{j} - 3\hat{k}$$

The direction ratios of the normal are 5, –2, –3. i.e. a = 5, b = – 2, c = – 3

And plane passes through A(1, –1, 0).

$\therefore$ Its Cartesian equation is a

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\therefore \quad 5(x - 1) - 2(y + 1) - 3(z - 0) = 0$$

$$\therefore \quad 5x - 2y - 3z - 7 = 0.$$

8) Find the vector equation of the plane passing through the point (1, 0. 2) and the line of intersection of planes $\bar{r}.(\hat{i}+\hat{j}+\hat{k}) = 8$ and $\bar{r}(2\hat{i}+3\hat{j}+4\hat{k}) = 3$.

Ans.: The equation of the required plane is of the form $\bar{r}.(\bar{n}_1 + \lambda\bar{n}_2) - (d_1 + \lambda d_2) = 0$

$$\therefore \bar{r}\left[(\hat{i}+\hat{j}+\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k})\right] = 8 + 3\lambda \; ... (1)$$

$$\therefore \bar{r}.\left((1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right) = 8 + 3\lambda$$

The plane passes through the point (1, 0, 2).

$$\therefore (\hat{i}+2\hat{k}).\left((1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right) = 8 + 3\lambda$$

$$\therefore (1+2\lambda) + 2(1+4\lambda) = 8 + 3\lambda$$

$$\therefore 1 + 2\lambda + 2 + 8\lambda = 8 + 3\lambda$$

$$\therefore 7\lambda = 5$$

$$\therefore \lambda = \frac{5}{7} \; ... (2)$$

From (1) and (2) we get

$$\therefore \bar{r}\left((\hat{i}+\hat{j}+\hat{k}) + \frac{5}{7}(2\hat{i}+3\hat{j}+4\hat{k})\right) = 8 + 3\left(\frac{5}{7}\right)$$

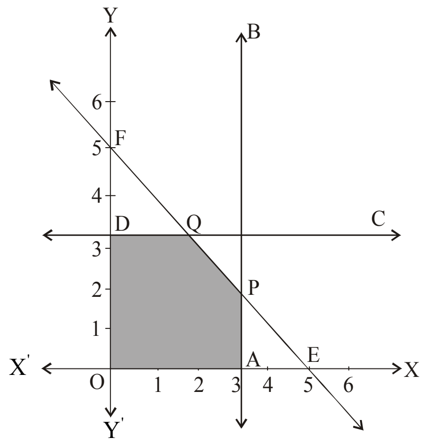
$$\therefore \bar{r}(17\hat{i} + 22\hat{j} + 27\hat{k}) = 71$$

9) Solve the following L.P.P. by graphical method

Maximize z =10x_1 + 25x_2 , Subject to $0 \le x_1 \le 3$, $0 \le x_2 \le 3$, $x_1 + x_2 \le 5$.

Ans.:

Inequalities	equalities	X-axis	Y-axis
$x_1 \le 3$	$x_1 = 3$	A(3.0)	–
$x_2 \le 3$	$x_2 = 3$	–	D(0,3)
$x_1 + x_2 \le 5$	$x + y = 5$	E(5,0)	F(0,5)



The feasible region is as shown in fig

The vertices of feasible region are O, A, P, Q, D Coordinate of P :

$x_1 = 3$ --------(1)

$x_1 + x_2 = 5$ --------(3)

solving (1) and (3),

$x_1 = 3$, $x_2 = 2$ $\therefore$ P = (3 , 2)

Coordinate of Q :

$x_2 = 3$ --------(2)

$x_1 + x_2 = 5$ --------(3)

solving (2) and (3),

$x_1 = 2$, $x_2 = 3$ $\therefore$ Q = (2 , 3)

$\therefore$ Coordinate of vertices are

O (0,0) , A(3,0) , P(3,2) , Q(2,3) , D(0,3)

Value of $z = 10x_1 + 25x_2$

Z(O) = 10(0) + 25(0) = 0 + 0 = 0

Z(A) = 10(3) + 25(0) = 30 + 0 = 21

Z(P) = 10(3) + 25(2) = 30 + 50 = 80

Z(Q) = 10(2) + 25(3) = 20 + 75 = 95

Z(B) = 10(0) + 25(3) = 0 + 75 = 75

$\therefore$ Maximum value of z = 95 at Q(2, 3)

OR

Find the vector equation of the line passing through the point $2\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$.

Ans.: Let $\bar{a} = 2\hat{i}+\hat{j}-3\hat{k}$, $\bar{b} = \hat{i}+\hat{j}+\hat{k}$ and $\bar{c} = \hat{i}+2\hat{j}-\hat{k}$

$$\bar{b}\times\bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}(-1-2) - \hat{j}(-1-1) + \hat{k}(2-1)$$

$$= -3\hat{i}+2\hat{j}+\hat{k}$$

The vector equation of the line is

$$\bar{r} = \bar{a} + \lambda\left(\bar{b}\times\bar{c}\right)$$

$$\therefore \quad \bar{r} = \left(2\hat{i}+\hat{j}-3\hat{k}\right) + \lambda\left(-3\hat{i}+2\hat{j}+\hat{k}\right)$$

Section - D (12 Marks)

10) Find the shortest distance between lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and}$$

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

Ans.: The vector equations of given lines are

$\bar{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and

$\bar{r} = (2\hat{i}+4\hat{j}+5\hat{k}) + \mu(3\hat{i}+4\hat{j}+5\hat{k})$

The shortest distance between lines $\bar{r} = \bar{a}_1 + \lambda\bar{b}_2$ and $\bar{r} = \bar{a}_2 + \lambda\bar{b}_2$ is

$$\left|\frac{(\bar{a}_2-\bar{a}_1).(\bar{b}_1\times\bar{b}_2)}{|\bar{b}_1\times\bar{b}_2|}\right| \text{ and}$$

Here $\bar{a}_1 = \hat{i}+2\hat{j}+3\hat{k}$, $\bar{a}_2 = 2\hat{i}+4\hat{j}+5\hat{k}$,

$\bar{b}_1 = 2\hat{i}+3\hat{j}+4\hat{k}, \bar{b}_2 = 3\hat{i}+4\hat{j}+5\hat{k})$

$\bar{a}_2 - \bar{a}_1 = (2\hat{i}+4\hat{j}+5\hat{k}) - (\hat{i}+2\hat{j}+3\hat{k})$

$= \hat{i}+2\hat{j}+2\hat{k})$

$$\text{And } \bar{b}_1\times\bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i}+2\hat{j}-\hat{k}$$

$$\left|\bar{b}_1\times\bar{b}_2\right| = \sqrt{1+4+1} = \sqrt{6}$$

$$(\bar{a}_2-\bar{a}_1).(\bar{b}_1\times\bar{b}_2) = (\hat{i}+2\hat{j}+2\hat{k}).$$

$$(-\hat{i}+2\hat{j}-\hat{k}) = 1$$

$\therefore$ The required shortest distance

$$\left|\frac{(\bar{a}_2-\bar{a}_1).(\bar{b}_1\times\bar{b}_2)}{\left|\bar{b}_1\times\bar{b}_2\right|}\right| = \frac{1}{\sqrt{6}} \text{ unit.}$$

11) Find the distance between the lines

$\bar{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ **and**

$$\bar{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$$

Ans.: By comparing with

$\bar{r} = \bar{a}_1 + \lambda\,\bar{b}$ and $\bar{r} = \bar{a}_2 + \mu\bar{b}$

We get,

$$\bar{a}_1 = \hat{i}+2\hat{j}-4\hat{k},\ \bar{a}_2 = 3\hat{i}+3\hat{j}-5\hat{k},\ \bar{b} = 2\hat{i}+3\hat{j}+6\hat{k}$$

$$\therefore\ \bar{a}_2 - \bar{a}_1 = 2\hat{i}+\hat{j}+\hat{k}$$

$$(\bar{a}_2 - \bar{a}_1)\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i}-14\hat{j}+4\hat{k}$$

Since lines are paraller

$\therefore$ Shortest distance

$$= \left|\frac{(\bar{a}_2 - \bar{a}_1)\times\bar{b}}{|\bar{b}|}\right| = \frac{\sqrt{293}}{7} \text{ units.}$$

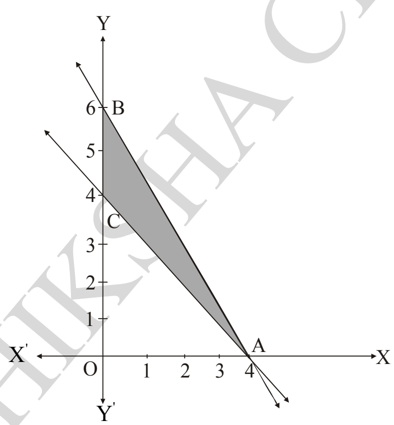
OR

Solve the following L.P.P. by graphical method

Maximize z = 4x + 6y, Subject to $3x + 2y \le 12$, $x + y \ge 4$, $x \ge 0$, $y \ge 0$.

Ans. :

Inequalities	equalities	X-axis	Y-axis
$3x+2y \le 12$	$3x+2y=12$	A(4,0)	B(0,6)
$x+y \ge 4$	$x+y=4$	C(4,0)	D(0,4)



The feasible region is as shown in fig.

The vertices of feasible region are A, B, C

$\therefore$ Coordinate of vertices are A(4,0) B(0,6) , C(0,4)

Value of z = 4x + 6y

Z(A) = 4(4) + 6(0) = 16 + 0 = 16

Z(B) = 4(0) + 6(6) = 0 + 36 = 36

Z(C) = 4(0) + 6(4) = 0 + 24 = 24

$\therefore$ Maximum value of z = 36 at B(0, 6)

12) If a line drawn from the pointA(1,2,1) is perpendicular to the line joining the points P(1,4,6) and Q(5,4,4),then find the coordinates of the foot of the perpendicular.

Ans. : Let AM be the perpendicular from the point A to the line PQ.

Let M divides PQ internally in the ratio k : 1

By internal sectional formula

$$\therefore\ M \equiv \left(\frac{5k+1}{k+1}, \frac{4k+4}{k+1}, \frac{4k+6}{k+1}\right)$$

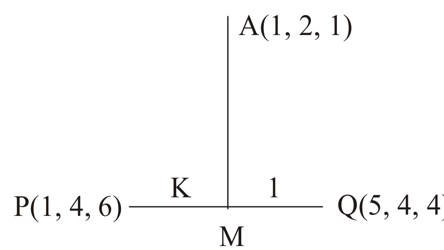
A(1, 2, 1) ,

$$M\left(\frac{5k+1}{k+1}, \frac{4k+4}{k+1}, \frac{4k+6}{k+1}\right)$$

The direction ratios of AM are

$$\frac{5k+1}{k+1}-1, \frac{4k+4}{k+1}-2, \frac{4k+6}{k+1}-1$$

$$\text{i.e. } \frac{4k}{k+1}, \frac{2k+2}{k+1}, \frac{3k+5}{k+1}$$



and the direction ratios of PQ are

5 – 1 , 4 – 4 , 4 – 6

i.e. 4 , 0 , –2

since AM $\perp$ PQ

$$\therefore\ 4\times\frac{4k}{k+1} + 0\times\frac{2k+2}{k+1} - 2\times\frac{3k+5}{k+1} = 0$$

$$\therefore\ \frac{16k}{k+1} - \frac{6k+10}{k+1} = 0$$

$$\therefore\ \frac{16k-6k-10}{k+1} = 0$$

$\therefore$ 10 k – 10 = 0

$\therefore$ 10 k = 10 $\quad\therefore$ k = 1

$$\therefore\ M \equiv \left(\frac{5k+1}{k+1}, \frac{4k+4}{k+1}, \frac{4k+6}{k+1}\right)$$

$$\therefore\ M \equiv \left(\frac{5(1)+1}{1+1}, \frac{4(1)+4}{1+1}, \frac{4(1)+6}{1+1}\right)$$

$$\therefore\ M \equiv \left(\frac{5+1}{2}, \frac{4+4}{2}, \frac{4+6}{2}\right)$$

$$\therefore\ M \equiv \left(\frac{6}{2}, \frac{8}{2}, \frac{10}{2}\right) = (3,4,5)$$

$$\therefore\ M \equiv (3,4,5)$$

Hence the coordinates of the foot of perpendicular are (3,4,5)

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