SHIKSHA CLASSES

Subject : Maths- II **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 6. Differential Equations**

Section - A (2 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) The solution of cosx.cosy dy – sinx.siny dx = 0 is

Ans:Option (a)

Hint: $\cos x.\cos y\,dy - \sin x.\sin y\,dx = 0$

$$\therefore \quad \frac{\cos y}{\sin y}dy - \frac{\sin x}{\cos x}dx = 0$$

$$\therefore \quad \cot y\,dy - \tan x\,dx = 0$$

On integrating

$$\therefore \quad \int \cot y\,dy - \int \tan x\,dx = 0$$

$\therefore \quad \log|\sin y| - \log|\sec x| = \log c$

$\therefore \quad \log|\sin y| + \log|\cos x| = \log c$

$\therefore \quad \log|\sin x.\sin y| = \log c$

$\therefore \quad \sin y.\cos x = c$

This is the general solution.

2) The differential equation obtained on eliminating A and B from y = A cos ωt + B Sin ωt is

Ans:Option(b)

Hint: $y = A\cos\omega t + B\sin\omega t$ ------ (i)

diff w.r.t.t.

$$\therefore \quad \frac{dy}{dt} = -\omega A\sin\omega t + \omega B\cos\omega t \text{ ---- (ii)}$$

again diff. w.r.t.t.

$$\frac{d^2y}{dt^2} = -\omega^2 A\cos\omega t - \omega^2 B\sin\omega t$$

$$= -\omega^2 (A\cos\omega t + B\sin\omega t)$$

$$\frac{d^2y}{dt^2} = -\omega^2 y \text{ ----- (from (i))}$$

$$\therefore \quad y'' = -\omega^2 y$$

Section - B (4 Marks)

3) Verify that y = logx + c is a solution of

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Ans.: $y = \log x + c$

Differentiate w.r.t. x , we get

$$\therefore \quad \frac{dy}{dx} = \frac{1}{x} \therefore x\frac{dy}{dx} = 1$$

Again differentiate w.r.t. x

$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = 0$$

Hence y = logx + c is a solution of

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

OR

Solve the differential equation:

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

Ans: $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$

$$\therefore \quad \frac{1}{1+y^2}dy = \frac{1}{1+x^2}dx$$

Integrating both sides

$$\int \frac{1}{1+y^2}dy = \int \frac{1}{1+x^2}dx$$

$\therefore \tan^{-1} y = \tan^{-1} x + c$

4) Obtain the differential equation by elliminating the arbitary constants y = A cos(logx) +B sin (logx)

Ans: y = A cos(logx) +B sin (logx)

Differentiate w.r.t. x , we get

$\therefore \frac{dy}{dx} = -A\sin(\log x).\frac{1}{x} + B\cos(\log x).\frac{1}{x}$

$\therefore x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$

Again differentiate w.r.t. x , we get

$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx}.1$

$= -A\cos(\log x).\frac{1}{x} - B\sin(\log x).\frac{1}{x}$

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$

$-[A\cos(\log x) + B\sin(\log x)]$

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Section - C (6 Marks)

5) Solve $\frac{dy}{dx} = (9x + y + 2)^2$

Ans. : $\frac{dy}{dx} = (9x + y + 2)^2$ ----(1)

put $9x + y + 2 = v$

$\therefore 9 + \frac{dy}{dx} = \frac{dv}{dx} \quad \therefore \frac{dy}{dx} = \frac{dv}{dx} - 9$

Equation (i) become

$\frac{dv}{dx} - 9 = v^2 \quad \therefore \frac{dv}{dx} = v^2 + 9$

$\therefore \frac{dv}{v^2+9} = dx \quad \therefore \int \frac{dv}{v^2+9} = \int dx$

$\therefore \frac{1}{3}\tan^{-1}\left(\frac{v}{3}\right) = x + c_1$

$\therefore \tan^{-1}\left(\frac{v}{3}\right) = 3x + 3c_1$

$\therefore \tan^{-1}\left(\frac{9x+y+2}{3}\right) = 3x + c$

OR

Form the differential equation by elliminating arbitary constant.

$y = c_1e^{3x} + c_2e^{-3x}$

Ans. : $y = c_1e^{3x} + c_2e^{-3x}$ ------ (1)

$\therefore \frac{dy}{dx} = 3c_1e^{3x} - 3c_2e^{-3x}$ ------ (2)

$\therefore \frac{d^2y}{dx^2} = 9c_1e^{3x} + 9c_2e^{3x}$ ------ (3)

$$\begin{vmatrix} y & 1 & 1 \\ \frac{dy}{dx} & 3 & -3 \\ \frac{d^2y}{dx^2} & 9 & 9 \end{vmatrix} = 0$$

$\therefore y(27+27) - 1\left(9\frac{dy}{dx} + 3\frac{d^2y}{dx^2}\right)$

$+1\left(9\frac{dy}{dx} - 3\frac{d^2y}{dx^2}\right) = 0$

$\therefore 54y - 9\frac{dy}{dx} - 3\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 3\frac{d^2y}{dx^2} = 0$

$= -6\frac{d^2y}{dx^2} + 54y = 0$

$\therefore 6\frac{d^2y}{dx^2} - 54y = 0$

6) Find the particular solution $\frac{dy}{dx} = 3^{x+y}$ when

x = 0, y = 0

Ans. : $\frac{dy}{dx}=3^{x+y}$

$\therefore \frac{dy}{dx}=3^x.3^y$

$\therefore 3^{-y}\,dy=3^x\,dx$

$\therefore \int 3^{-y}\,dy=\int 3^x\,dx$

$\therefore \int 3^x\,dx-\int 3^{-y}\,dy=0$

$\therefore \frac{3^x}{\log 3}-\frac{3^{-y}}{-1\log 3}=c_1 \therefore \frac{3^x}{\log 3}+\frac{3^{-y}}{\log 3}=c_1$

$\therefore 3^x+3^{-y}=c_1.\log 3=C.$

put x= 0 and y = 0

$\therefore 1+1=c \quad \therefore c=2$

The particular solution is $\therefore 3^x+3^{-y}=2$

Section - D (8 Marks)

7) Solve

$: \frac{y}{x}\cos\left(\frac{y}{x}\right)\left(\frac{dy}{dx}-\frac{y}{x}\right)+\sin\left(\frac{y}{x}\right)\left(\frac{dy}{dx}+\frac{y}{x}\right)=0$

when x = 1 and $y=\frac{\pi}{2}$

Ans. : $\frac{y}{x}\cos\frac{y}{x}\left(\frac{dy}{dx}-\frac{y}{x}\right)+\sin\frac{y}{x}\left(\frac{dy}{dx}+\frac{y}{x}\right)=0$

put $y=ux \therefore \frac{dy}{dx}=u+x\frac{du}{dx}$

$\therefore u.\cos u\left(u+x\frac{du}{dx}-u\right)+\sin u\left(u+x\frac{du}{dx}+u\right)=0$

$xu\cos u\frac{du}{dx}+2u\sin u+x\sin u.\frac{du}{dx}=0$

$\therefore x\left(u\cos u+\sin u\right)\frac{dy}{dx}=-2u\sin u$

$\therefore \int \frac{u\cos u+\sin u}{u\sin u}du=-2\int \frac{dx}{x}$

$\therefore \log\left(u.\sin u\right)=-2\log x+\log c$

$\therefore \log\left(u.\sin u\right)+\log x^2=\log c$

$\therefore \log\left[u.\sin u.x^2\right]=\log c$

$u.\left(\sin u\right)x^2=c \Rightarrow \frac{y}{x}\left(\sin\frac{y}{x}\right)x^2=c$

$xy\sin\frac{y}{x}=c \Rightarrow \frac{y}{x}\left(\sin\frac{y}{x}\right)x^2=c$

$\therefore xy\sin\frac{y}{x}=c \Rightarrow$ where $x=1$ and $u=\frac{\pi}{2}$

$\therefore \frac{\pi}{2}\sin\frac{\pi}{2}=c \Rightarrow c=\frac{\pi}{2}$

$\therefore y\sin\frac{y}{x}=\frac{\pi}{2x}$

8) Solve the differential equation $x^2dy+y(x+y)\,dx=0$

Ans.: $x^2dy=-(xy+y^2)dx$

$\frac{dy}{dx}=-\frac{xy+y^2}{x^2}$

Put $y=ux \Rightarrow \frac{dy}{dx}=u+x\frac{du}{dx}$

$\therefore u+x\frac{du}{dx}=-\frac{ux^2+u^2x^2}{x^2}=-u-u^2$

$x\frac{du}{dx}=-u^2-2u \Rightarrow \int \frac{du}{u(u+2)}=-\int \frac{dx}{x}$

$\int \frac{2+u-u}{u(u+2)}du=-2\int \frac{dx}{x}$

$\int \left[\frac{1}{u}-\frac{1}{u+2}\right]du=-2\int \frac{dx}{x}$

$\log u-\log(u+2)=-2\log x+\log c$

$\log\left(\frac{u}{u+2}\right)+\log x^2=\log c$

i.e. $\left(\frac{u}{u+2}\right)x^2 = c$

$$\frac{y}{x\left(\frac{y}{x}+2\right)}x^2 = c \Rightarrow \frac{yx^2}{y+2x} = c$$

$\therefore \quad yx^2 = c(y+2x)$

$\therefore \quad x^2y = c(2x+y)$

OR

The rate of growth of the population of a city at any time t is proportional to the size of the population. For a certain city it is found that the constant of propor tionality is 0.04. Find the population of the city after 25 years if the initial population is 10,000.

[Take e = 2.7182]

Ans.: Let P be the population at time t yrs.

$\therefore \frac{dp}{dt} \alpha$ P $\therefore \frac{dp}{dt} =$ kP, where k = 0.04

$\therefore \frac{dp}{dt} =$ (0.04)P $\quad \therefore \frac{dp}{P} = (0.04)$ dt

On integrating, we get,

$$\int \frac{dp}{P} = 0.04 \int dt + c$$

$\therefore$ log P = 0.04t + c

when t = 0, let P = 10000

$\therefore$ log 10000= (0.04) $\times$ 0 + c

$\therefore$ c = log 10000

$\therefore$ log P = (0.04)t + log 10000

$\therefore$ log P – log10000 = (0.04)t

$\therefore \quad \log\left(\frac{P}{10000}\right) = (0.04)t \qquad ...(1)$

when t = 25 , P = ?

$\therefore \quad \log\left(\frac{P}{10000}\right) = (0.04)25$

$\therefore \quad \log\left(\frac{P}{10000}\right) = 1$

$\therefore \quad \log\left(\frac{P}{10000}\right)$ =loge $\therefore \frac{P}{10000} = e$

$\therefore$ P =10000(2.7182) $\therefore$ P = 27182

$\therefore$ The population of the city at any time t is 27182

* * *


BECOME AN ACE IN JEE & NEET
SHIKSHA CLASSES
Believe & Achieve
JEE | NEET | Previse (8-10)
8625055707 | 8623085707
shikshaclasses.co.in
M-19, MHADA Colony, Khat Road, Bhandara
Learn with Jaiswal sir