SHIKSHA CLASSES

Subject : Maths- I **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 5. Vector**

Section - A (2 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) If the vectors $2\hat{i}-3\hat{j}$, $\hat{i}+\hat{j}-\hat{k}$ and $3\hat{i}-\hat{k}$ form the three concurrent edges of a parallelopiped then the volume of the parallelopiped is

Ans: (C) : 4

$\bar{a}=2\hat{i}-3\hat{j}$, $\bar{b}=\hat{i}+\hat{j}-\hat{k}$, $\bar{c}=3\hat{i}-\hat{k}$

Volume of the parallelopiped $=\bar{a}.\bar{b}\times\bar{c}$

$$=\begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$= 2(-1+0) + 3(-1+3) + 0(0-3)$

$= 2(-1) + 3(2) + 0(-3)$

$= -2 + 6 - 0 = 4$

2) The coordinates of the points which divides the line joining the point P (2, – 1, – 4) and Q (3, – 2, 5) externally in the ratio 2 : 3 is

Ans: (D) (0, 1, -22)

$\bar{p}=2\hat{i}-\hat{j}-4\hat{k}$, $\bar{q}=3\hat{i}-2\hat{j}+5\hat{k}$

m : n = 2 : 3 $\therefore$ m = 2, n = 3.

By external division sectional formula

$$\bar{r}=\frac{m.\bar{q}-n.\bar{p}}{m-n}$$

$$=\frac{2\left(3\hat{i}-2\hat{j}+5\hat{k}\right)-3\left(2\hat{i}-\hat{j}-4\hat{k}\right)}{2-3}$$

$$=\frac{6\hat{i}-4\hat{j}+10\hat{k}-6\hat{i}+3\hat{j}+12\hat{k}}{-1}$$

$$=\frac{-\hat{j}+22\hat{k}}{-1} \quad \bar{r}=\hat{j}-22\hat{k}$$

$\therefore$ R $\equiv$ (0, 1, -22)

Section - B (4 Marks)

3) Show that the points A $\equiv$ (2,1,1), B $\equiv$ (0,–1, 4), C $\equiv$ (4,3,–2) are collinear.

Ans : $\bar{a}=2\hat{i}+\hat{j}+\hat{k}$, $\bar{b}=-\hat{j}+4\hat{k}$

$\bar{c}=4\hat{i}+3\hat{j}-2\hat{k}$

$\overline{AB}=\bar{b}-\bar{a}=-\hat{j}+4\hat{k}-\left(2\hat{i}+\hat{j}+\hat{k}\right)$

$=-\hat{j}+4\hat{k}-2\hat{i}-\hat{j}-\hat{k}$

$=-2\hat{i}-2\hat{j}+3\hat{k}$

$\therefore \overline{AB}=-2\hat{i}-2\hat{j}+3\hat{k}$

$\overline{AC}=\bar{c}-\bar{a}=4\hat{i}+3\hat{j}-2\hat{k}-\left(2\hat{i}+\hat{j}+\hat{k}\right)$

$=4\hat{i}+3\hat{j}-2\hat{k}-2\hat{i}-\hat{j}-\hat{k}$

$=2\hat{i}+2\hat{j}-3\hat{k}$

$\therefore \overline{AC}=2\hat{i}+2\hat{j}-3\hat{k}$

$$\therefore \frac{\overline{AB}}{\overline{AC}}=\frac{-2\hat{i}-2\hat{j}+3\hat{k}}{2\hat{i}+2\hat{j}-3\hat{k}}=\frac{-\left(2\hat{i}+2\hat{j}-3\hat{k}\right)}{2\hat{i}+2\hat{j}-3\hat{k}}$$

$\therefore \frac{\overline{AB}}{\overline{AC}}=-1 \quad \therefore \overline{AB}=(-1)\overline{AC}$

$\therefore$ $\overline{AB}$ is scalar multiple of $\overline{AC}$

$\therefore$ $\overline{AB}$ and $\overline{AC}$ are collinear vectors.

But A is common point.

$\therefore$ The point A,B and C are collinear.

OR

If two vertices of the triangle are A(3, 1, 4) and B (– 4, 5, – 3), and the centroid of the triangle is at G(–1,2, 1) then find the

coordinates of the third vertex C of the triangle.

Ans: Let $\bar{a},\bar{b},\bar{c}$ be the position vectors of points A,B,C respectively

Then $\bar{a} = 3\hat{i} + \hat{j} + 4\hat{k}$

$\bar{b} = -4\hat{i} + 5\hat{j} - 3\hat{k}$

$\bar{g} = -\hat{i} + 2\hat{j} + \hat{k}$

By centroid formula

$\bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3} \quad \therefore\ 3\bar{g} = \bar{a}+\bar{b}+\bar{c}$

$\therefore\ 3\left(-\hat{i}+2\hat{j}+\hat{k}\right) = 3\hat{i}+\hat{j}+4\hat{k}-4\hat{i}+5\hat{j}-3\hat{k}+\bar{c}$

$\therefore\ -3\hat{i}+6\hat{j}+3\hat{k} = -\hat{i}+6\hat{j}+\hat{k}+\bar{c}$

$\therefore\ \bar{c} = -3\hat{i}+6\hat{j}+3\hat{k}+\hat{i}-6\hat{j}-\hat{k}$

$\therefore\ \bar{c} = -2\hat{i}+0\hat{j}+2\hat{k} \quad \therefore$ C (–2,0,2)

4) If A, B, C, D are four non-collinear points in the plane such that $\overline{AD}+\overline{BD}+\overline{CD}=0$, then prove that the point D is the centroid of the triangle ABC.

Ans: Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ be the position vectors of the points A, B, C, ,D respectively.

$\overline{AD}+\overline{BD}+\overline{CD}=0$

$\therefore\ \bar{d}-\bar{a}+\bar{d}-\bar{b}+\bar{d}-\bar{c}=0$

$\therefore\ 3\bar{d}-\bar{a}-\bar{b}-\bar{c}=0$

$\therefore\ 3\bar{d}=\bar{a}+\bar{b}+\bar{c}$

$\therefore\ \bar{d}=\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Hence D is the centroid of the Δ ABC.

Section - C (6 Marks)

5) If $\bar{a}=\hat{i}-\hat{j}+4\hat{k}$, $\bar{b}=\hat{i}+\hat{j}-4\hat{k}$, $\bar{c}=\hat{i}+\hat{j}+\hat{k}$ find $\bar{a}.\left(\bar{b}\times\bar{c}\right)$

Ans : $\bar{a}=\hat{i}-\hat{j}+4\hat{k}$, $\bar{b}=\hat{i}+\hat{j}-4\hat{k}$, $\bar{c}=\hat{i}+\hat{j}+\hat{k}$

$$\bar{a}.\left(\bar{b}\times\bar{c}\right)=\begin{vmatrix}1 & -1 & 4\\ 1 & 1 & -4\\ 1 & 1 & 1\end{vmatrix}$$

$= 1(1+4) + 1(1+4) - 4(1-1)$

$= 1(5) + 1(5) - 4(0)$

$= 5 + 5 - 0 = 10$

OR

If G_1 and G_2 are the centroids of ΔABC and ΔPQR respectively then show that $\overline{AP}+\overline{BQ}+\overline{CR}=3\overline{G_1G_2}$

Ans : Let $\bar{a},\bar{b},\bar{c},\bar{p},\bar{q},\bar{r},\bar{g_1},\bar{g_2}$ be the position vectors of the points A,B,C,P, Q, R,G_1 and G_2 respectively.

G_1 is the centroid of ΔABC

By centroid formula

$\therefore\ \bar{g}_1=\dfrac{\bar{a}+\bar{b}+\bar{c}}{3} \quad \therefore\ 3\bar{g}_1=\bar{a}+\bar{b}+\bar{c} \quad$ ...(i)

G_2 is the centroid of ΔPQR

$\therefore\ \bar{g}_2=\dfrac{\bar{p}+\bar{q}+\bar{r}}{3} \quad \therefore 3\bar{g}_2=\bar{p}+\bar{q}+\bar{r} \quad$ ...(ii)

L.H.S. = $\overline{AP}+\overline{BQ}+\overline{CR}$

$= \bar{p}-\bar{a}+\bar{q}-\bar{b}+\bar{r}-\bar{c}$

$= \left(\bar{p}+\bar{q}+\bar{r}\right)-\left(\bar{a}+\bar{b}+\bar{c}\right)$

$= 3\bar{g}_2-3\bar{g}_1 \quad [\because$ From (i) and (ii)]

$= 3\left(\bar{g}_2-\bar{g}_1\right) = 3\overline{G_1G_2}$

L.H.S. = R.H.S.

6) Find the volume of tetrahedron whose vertices are A(3,7,4),B(5,–2,3),C(– 4,5,6) and D(1,2, 3).

Ans : Let $\bar{a}=3\hat{i}+7\hat{j}+4\hat{k}$, $\bar{b}=5\hat{i}-2\hat{j}+3\hat{k}$,

$\bar{c}=-4\hat{i}+5\hat{j}+6\hat{k}$, $\bar{d}=\hat{i}+2\hat{j}+3\hat{k}$

$\overline{AB}=\bar{b}-\bar{a}=5\hat{i}-2\hat{j}+3\hat{k}-\left(3\hat{i}+7\hat{j}+4\hat{k}\right)$

$=5\hat{i}-2\hat{j}+3\hat{k}-3\hat{i}-7\hat{j}-4\hat{k}$

$=2\hat{i}-9\hat{j}-\hat{k}$

$\overline{AC}=\bar{c}-\bar{a}=-4\hat{i}+5\hat{j}+6\hat{k}-\left(3\hat{i}+7\hat{j}+4\hat{k}\right)$

$=-4\hat{i}+5\hat{j}+6\hat{k}-3\hat{i}-7\hat{j}-4\hat{k}$

$$=-7\hat{i}-2\hat{j}+2\hat{k}$$

$$\overline{AD}=\bar{d}-\bar{a}=\hat{i}+2\hat{j}+3\hat{k}-\left(3\hat{i}+7\hat{j}+4\hat{k}\right)$$

$$=\hat{i}+2\hat{j}+3\hat{k}-3\hat{i}-7\hat{j}-4\hat{k}$$

$$=-2\hat{i}-5\hat{j}-\hat{k}$$

Volume of tetrahedron $=\frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$$=\frac{1}{6}\begin{vmatrix}2 & -9 & -1\\ -7 & -2 & 2\\ -2 & -5 & -1\end{vmatrix}$$

$$=\frac{1}{6}[2\ (2+10)+9(7+4)-1(35-4)]$$

$$=\frac{1}{6}[2(12)+9(11)-1(31)]$$

$$=\frac{1}{6}[24+99-31]=\frac{1}{6}(92)=\frac{46}{3}\text{ cu.units}$$

Section - D (8 Marks)

7) If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\bar{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then prove that

$$\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

Ans: $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

$$\bar{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$$

$$\bar{b}\times\bar{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$$=\hat{i}(b_2c_3-b_3c_2)-\hat{j}(b_1c_3-b_3c_1)+\hat{k}(b_1c_2-b_2c_1)$$

$$\bar{a}.\bar{b}\times\bar{c}=$$

$$a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$$

$$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$$\therefore \bar{a}.\bar{b}\times\bar{c}=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

8) If P and Q are any two points having position vectors $\bar{p}$ and $\bar{q}$ with respect to O and R divides seg PQ externally in the ratio m : n then prove that $\bar{r}=\frac{m\bar{q}-n\bar{p}}{m-n}$, where $\bar{r}$ is the position vector of R.

Ans :

Let $\bar{p},\bar{q},\bar{r}$ be the position vectors of points P , Q and R respectively

since $\frac{\ell(PR)}{\ell(RQ)}=\frac{m}{n}$

$\therefore$ n ℓ (PR) = m ℓ (RQ)

But the direction of PR and RQ is the opposite

$\therefore$ n $\overline{PR}=-$ m $\overline{RQ}$

$$\therefore n\left(\bar{r}-\bar{p}\right)=-m\left(\bar{q}-\bar{r}\right)$$

$$\therefore n\,\bar{r}-n\,\bar{p}=-m\,\bar{q}+m\,\bar{r}$$

$$\therefore -m\,\bar{r}+n\,\bar{r}=-m\,\bar{q}+n\,\bar{p}$$

$$\therefore m\,\bar{r}-n\,\bar{r}=m\,\bar{q}-n\,\bar{p}$$

$$\therefore (m-n)\bar{r}=m\bar{q}-n\bar{p}$$

$$\therefore \bar{r}=\frac{m\ \bar{q}-n\ \bar{p}}{m-n}$$

OR

If A (2, 3, -1), B (-2, -3, -3), C (1, 7, 2), D (-6, 2, 2) are four Points such that $\overline{AB}=t_1\overline{AC}+t_2\overline{AD}$ then find values of t_1 & t_2

Ans : Let $\overline{AB}=t_1\overline{AC}+t_2\overline{AD}$ _____ (1)

Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are position vectors of the points A, B, C & D respectively But
A (2, 3, -1), B (-2, -3, -3)
C (1, 7, 2) & D (-6, 2, 2)

$\therefore \bar{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \bar{b} = -2\hat{i} - 3\hat{j} - 3\hat{k}, \bar{c}$

$= \hat{i} + 7\hat{j} + 2\hat{k}, \bar{d} = \bar{6}\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \overline{AB} = \bar{b} - \bar{a}$

$= (-2\hat{i} - 3\hat{j} - 3\hat{k}) - (2\hat{i} + 3\hat{j} - \hat{k})$

$\overline{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$ ....(2)

$\overline{AC} = \bar{c} - \bar{a} = (\hat{i} + 7\hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} - \hat{k})$

$\overline{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$ _______(3)

& $\overline{AD} = \bar{d} - \bar{a} = (-6\hat{i} + 2\hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} - \hat{k})$

$= -8\hat{i} - \hat{j} + 3\hat{k}$

From (1) :

$\overline{AB} = t_1 \overline{AC} + t_2 \overline{AD}$

$\therefore$ *From* eq^n (1), (2) & (3)

$\therefore -4\hat{i} - 6\hat{i} - 2\hat{k} = t_1(-\hat{i} + 4\hat{j} + 3\hat{k}) + t_2(-8\hat{i} - \hat{j} + 3\hat{k})$

$\therefore -4\hat{i} - 6\hat{j} - 2\hat{k} = (-t_1 - 8t_2)\hat{i} + (4t_1 - t_2)\hat{j} + (3t_1 + 3t_2)\hat{k}$

$\therefore -t_1 - 8t_2 = -4$ ______ (4)

$4t_1 - t_2 = -6$ _______ (5)

$3t_1 + 3t_2 = -2$ ______(6)

eq^n (4) × 4 + eq^n (5)

$$\begin{array}{r} -4t_1 - 32t_2 = -16 \\ + \quad 4t_1 - t_2 = -6 \\ \hline -33t_2 = -22 \end{array}$$

$\therefore t_2 = \frac{-22}{-33} = \frac{2}{3}$

$\therefore t_2 = \frac{2}{3}$ putting this value in eq^n

(6) we get.

$3t_1 + 3 \times \frac{2}{3} = -2$

$\therefore 3t_1 = -2 - 2$

$\therefore 3t_1 = -4$

$\therefore t_1 = \frac{-4}{3}$

$\therefore \boxed{t_1 = \frac{-4}{3} \,\&\, t_2 = \frac{2}{3}}$

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