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Subject : Math -I **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 4. Pair of Straight Line**

[Section - A (2 Marks)]

Select and write the correct answer from the given alternatives for each of the following :

1) The value of k if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$ is

Ans. : c) 5

slope of line $2x + y = 0$ is $m = -2$

$6x^2 + kxy + y^2 = 0$ dividing by x^2

$\therefore \quad 6 + k\frac{y}{x} + \frac{y^2}{x^2} = 0$ put $\frac{y}{x} = m$

$\therefore \quad 6 + km + m^2 = 0$ .... (1)

Put m = –2 in equation (1)

$\therefore \quad 6 + k(-2) + (-2)^2 = 0$

$\therefore \quad 6 - 2k + 4 = 0$

$\therefore \quad -2k + 10 = 0 \qquad \therefore \quad -2k = -10$

$\therefore \quad 2k = 10 \qquad \therefore \quad k = 5$

2) The equation of the lines passing through the point (1,2)and parallel to coordinate axes.

Ans. : d) $xy - y - 2x + 2 = 0$

Let l_1 and l_2 be the lines parallel to X and Y axes equation of l_1 is $y = 2$ i.e. $y - 2 = 0$ equation of l_2 is $x = 1$ i.e. $x - 1 = 0$

The combined equation of l_1 and l_2 is

$(y - 2)(x - 1) = 0$

$\therefore \quad xy - y - 2x + 2 = 0$

[Section - B (4 Marks)]

3) Find the joint equation of the lines passing through the origin and having inclinations 60^0 and 120^0 with the X-axis.

Ans. : Let l_1 and l_2 be the two required lines.

Slope of the line l_1 is $m_1 = \tan 60^0 = \sqrt{3}$

The equation of l_1 is $y = \sqrt{3}x$

$\therefore \quad \sqrt{3}x - y = 0$ .... (i)

Slope of the line l_2 is $m_2 = \tan 120^0$

$= \tan(90^0 + 30^0)$

$= -\cot 30^0 = -\sqrt{3}$

The equation of l_2 is $y = -\sqrt{3}x$

$\therefore \quad \sqrt{3}x + y = 0$ .... (ii)

The joint equations of lines l_1 and l_2 is

$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x + y\right) = 0$

$\therefore \quad 3x^2 - y^2 = 0$

OR

Find k if the slopes of the lines represented by $kx^2 + 5xy + y^2 = 0$ differ by 1.

Ans. : The given homogeneous equations is

$kx^2 + 5xy + y^2 = 0$ .... (i)

Comparing with $ax^2 + 2hxy + by^2 = 0$

$a = k,\ 2h = 5 \ \therefore h = \frac{5}{2},\ b = 1$

let m_1 and m_2 are slopes of the lines

$m_1 + m_2 = \frac{-2h}{b} = \frac{-2 \times \frac{5}{2}}{-1} = 5,$

$m_1 \ m_2 = \frac{a}{b} = \frac{k}{1} = k.$

But $m_1 - m_2 = 1$ ..... (given)

we know that

$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m.m_2$

$\therefore \quad (1)^2 = (5)^2 - 4(k) \ \therefore 4k = 24 \ \therefore k = 6$

4) Find the joint equation of the lines $2x + y + 1 = 0$ and $2x - y - 1 = 0$

Ans. : The joint equation is

$(2x + y + 1)(2x - y - 1) = 0$

$4x^2 - 2xy - 2x + 2xy - y^2 - y + 2x - y - 1 = 0$

$\therefore \quad 4x^2 - y^2 - 2y - 1 = 0$

[Section - C (6 Marks)]

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5) Find the seperate equation of
$2x^2 + 7xy + 3y^2 = 0$

Ans. : $2x^2 + 7xy + 3y^2 = 0$

$\therefore\ 2x^2 + 6xy + xy + 3y^2 = 0$

$\therefore\ 2x(x + 3y) + y(x + 3y) = 0$

$\therefore\ (x + 3y)(2x + 3y) = 0$

$\therefore\ x + 3y = 0,\ 2x + 3y = 0$

These are the seperate equations.

OR

The slope of one of the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ is five times the other, show that $5h^2 = 9ab$.

Ans. :

The given homogeneous equation in second degree is

$ax^2 + 2hxy + by^2 = 0$ .... (i)

Let m_1, m_2 be the slopes of the lines represented by equation (i)

Given $m_1 = 5m_2$

$$\therefore\ m_1 + m_2 = \frac{-2h}{b},\quad m_1.m_2 = \frac{a}{b}$$

put $m_1 = 5m_2$

$$\therefore\ 5m_2 + m_2 = \frac{-2h}{b},\quad 5m_2.m_2 = \frac{a}{b}$$

$$\therefore\ 6m_2 = \frac{-2h}{b},\quad 5m_2^2 = \frac{a}{b}$$

$$\therefore\ m_2 = \frac{-2h}{6b}\ \ .... (1),\quad m_2^2 = \frac{a}{5b}\ .....(2)$$

from (1) and (2)

$$\therefore\ \frac{4h^2}{36b^2} = \frac{a}{5b}\quad \therefore\ \frac{h^2}{9b} = \frac{a}{5}$$

$\therefore\ 5h^2 = 9ab$

Hence proved.

6) If one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is px – qy = 0 show that $aq^2 + 2hpq + bp^2 = 0$

Ans. : Slope of line $px - qy = 0$ is $m = p/q$

$ax^2 + 2hxy + by^2 = 0$ dividing by x^2

$$\therefore\ a + 2h\frac{y}{x} + b\frac{y^2}{x^2} = 0 \quad \text{put } \frac{y}{x} = m$$

$\therefore\ a + 2hm + m^2 = 0$ .... (1)

put $m = p/q$ in equation (1)

$$\therefore\ a + 2h\left(\frac{p}{q}\right) + b\left(\frac{p}{q}\right)^2 = 0$$

$$\therefore\ a + \frac{2hp}{q} + \frac{bp^2}{q^2} = 0$$

$\therefore\ aq^2 + 2hpq + bp^2 = 0$ Hence proved.

[Section - D (8 Marks)]

7) Find the joint equation of pair of lines passing through the origin and perpendicular to the lines given by $5x^2 + 2xy - 3y^2 = 0$

Ans. : $5x^2 + 2xy - 3y^2 = 0$

comparing with

$ax^2 + 2hxy + by^2 = 0$

$a = 5,\ 2h = 2\ \therefore\ h = 1,\ b = -3$

$$\therefore\ m_1 + m_2 = \frac{-2h}{b} = \frac{-2(1)}{-3} = \frac{2}{3}$$

$$\therefore\ m_1.m_2 = \frac{a}{b} = \frac{5}{-3} = -\frac{5}{3}$$

Let l_1 and l_2 be the required lines.

$$\therefore\ \text{slope of line } l_1 = -\frac{1}{m_1}$$

$$\therefore\ \text{equation of } l_1 \text{ is } y = -\frac{1}{m_1}x$$

$\therefore\ m_1 y = -x \quad \therefore\ x + m_1 y = 0$

$$\text{slope of line } l_2 = -\frac{1}{m_2}$$

$$\therefore\ \text{equation of } l_2 \text{ is } y = -\frac{1}{m_2}x$$

$\therefore\ m_2 y = -x \quad \therefore\ x + m_2 y = 0$

combined equation of l_1 and l_2 is

$(x + m_1 y)(x + m_2 y) = 0$

$\therefore\ x^2 + m_2 xy + m_1 xy + m_1 m_2 y^2 = 0$

$\therefore\ x^2 + (m_1 + m_2)xy + m_1 m_2 y^2 = 0$

$$\therefore\ x^2 + \left(\frac{2}{3}\right)xy + \left(-\frac{5}{3}\right)y^2 = 0$$

$\therefore\ 3x^2 + 2xy - 5y^2 = 0$

8) Show that the acute angle between the lines

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$ax^2 + 2hxy + by^2 = 0$ **is**

$$\theta = \tan^{-1}\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

Hence find the condition that lines are

i) coincident

ii) perpendicular

Ans. :

The given homogenous equation in seocnd degree is $ax^2 + 2hxy + by^2 = 0$ .... (i)

Let m_1 and m_2 be the slopes of lines (i)

$$\therefore \quad m_1 + m_2 = -\frac{2h}{b},\ m_1 m_2 = \frac{a}{b}$$

$$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$$

$$= \left(\frac{-2h}{b}\right)^2 - 4.\frac{a}{b}$$

$$= \frac{4h^2}{b^2} - \frac{4a}{b} = \frac{4h^2 - 4ab}{b^2}$$

$$\therefore \quad |m_1 - m_2| = \left|\frac{2\sqrt{h^2 - ab}}{b}\right|$$

If θ is the acute angle between the lines then.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right| = \left|\frac{\frac{2\sqrt{h^2 - ab}}{b}}{1 + \frac{a}{b}}\right|$$

$$= \left|\frac{\frac{2\sqrt{h^2 - ab}}{b}}{\frac{a+b}{b}}\right| = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad \theta = \tan^{-1}\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

i) If lines are coincident then $\theta = 0$

$$\tan 0 = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad 0 = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad 2\sqrt{h^2 - ab} = 0$$

$$\therefore \quad h^2 - ab = 0$$

ii) If lines are perpendicular then $\theta = 90^0$.

$$\therefore \quad \tan 90 = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad \infty = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$\therefore \quad a + b = 0$$

OR

If the angle between lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between lines represented by $2x^2 - 5xy + 3y^2 = 0$ then show that $100(h^2 - ab) = (a + b)^2$.

Ans : Let 0 is angle between the lines $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$

$$\therefore 1 = 2, 2h = -5, b = 3$$

$$\therefore h = -5/2$$

$$\therefore h^2 - ab = \left(\frac{5}{2}\right)^2 - 2\times 3$$

$$= \frac{25}{4} - \frac{6}{1} = \frac{25 - 24}{4} = \frac{1}{4}$$

$$\therefore \sqrt{h^2 - ab} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\times\frac{1}{2}}{2+3}\right|$$

$$\therefore \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{1}{5}$$

$$10\sqrt{h^2 - ab} = a + b$$

$$\therefore \boxed{100\left(h^2 - ab\right) = \left(a + b\right)^2}$$

Hence proved.

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