SHIKSHA CLASSES

Subject : Math -II | **BOARD ANSWER PAPER** | **Total Marks : 20**

Class : XII | **Topic: 3. Indefinite Integration**

Section - A (2 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) $\displaystyle\int \frac{x^2}{x^2+4}dx =$

Ans. a) x – 2 tan^{-1} (x/2) +c

$$I=\int\frac{x^2}{x^2+4}dx=\int\frac{x^2+4-4}{x^2+4}dx$$

$$=\int 1-\frac{4}{x^2+4}dx = x-4\int\frac{1}{x^2+(2)^2}dx$$

$$=x-4\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$=x-2\tan^{-1}\left(\frac{x}{2}\right)+c$$

2) $\displaystyle\int\frac{dx}{x^3(1-x)}=$ _____

Ans. d) $\log\left|\frac{1-x}{x}\right|+\frac{1}{x}+\frac{1}{2x^2}+c$

$$I=\int\frac{1x}{x^3(1-x)}=\int\frac{1-x^3+x^3}{x^3(1-x)}dx$$

$$\int\left[\frac{(1-x)(1+x+x^2)}{x^3(1-x)}+\frac{1}{1-x}\right]dx$$

$$\int\left[\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}+\frac{1}{1-x}\right]dx$$

$$=\frac{-1}{2x^2}-\frac{1}{x}+\log x-\log|1-x|+c$$

$$=\log\left|\frac{x}{1-x}\right|-\frac{1}{x}-\frac{1}{2x^2}+c$$

Section - B (4 Marks)

3) Evaluate $\displaystyle\int\frac{1}{9x^2+6x+10}dx$

Ans. : Let $I=\displaystyle\int\frac{1}{9x^2+6x+10}dx$

$$=\frac{1}{9}\int\frac{1}{x^2+\frac{2}{3}x+\frac{10}{9}}dx$$

$$=\frac{1}{9}\int\frac{1}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{10}{9}}dx$$

$$=\frac{1}{9}\int\frac{1}{\left(x+\frac{1}{3}\right)^2+1}dx$$

$$=\frac{1}{9}\int\frac{1}{\left(x+\frac{1}{3}\right)^2+(1)^2}dx$$

$$=\frac{1}{9}.\frac{1}{1}\tan^{-1}\left(\frac{x+\frac{1}{3}}{1}\right)+c$$

$$=\frac{1}{9}\tan^{-1}\left(\frac{3x+1}{3}\right)+c$$

OR

Evaluate $\int \frac{1}{\cos\alpha+\cos x}dx$

Ans. : Let $I=\int \frac{1}{\cos\alpha+\cos x}dx$

put $t=\tan\left(\frac{x}{2}\right)$, $dx=\frac{2dt}{1+t^2}$,

$\cos x=\frac{1-t^2}{1+t^2}$

$$\therefore \quad I=\int \frac{1}{\cos\alpha+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$$

$$=\int \frac{1}{\frac{(1+t^2)\cos\alpha+(1-t^2)}{1+t^2}}\cdot\frac{2dt}{1+t^2}$$

$$=2\int \frac{1}{(1+t^2)\cos\alpha+1-t^2}dt$$

$$=2\int \frac{1}{\cos\alpha+t^2\cos\alpha+1-t^2}dt$$

$$=2\int \frac{1}{(1+\cos\alpha)-(1-\cos\alpha)t^2}dt$$

$$=2\int \frac{1}{2\cos^2\left(\frac{\alpha}{2}\right)-2\sin^2\left(\frac{\alpha}{2}\right)t^2}dt$$

$$=\int \frac{1}{\cos^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)t^2}dt$$

$$=\frac{1}{\sin^2\left(\frac{\alpha}{2}\right)}\int \frac{1}{\cot^2\left(\frac{\alpha}{2}\right)-t^2}dt$$

$$=co\sec^2\left(\frac{\alpha}{2}\right).\frac{1}{2\cot\left(\frac{\alpha}{2}\right)}\log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+t}{\cot\left(\frac{\alpha}{2}\right)-t}\right|+c$$

$$=\frac{1}{\sin^2\left(\frac{\alpha}{2}\right)}.\frac{\sin\left(\frac{\alpha}{2}\right)}{2\cos\left(\frac{\alpha}{2}\right)}\log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+t}{\cot\left(\frac{\alpha}{2}\right)-t}\right|+c$$

$$=\frac{1}{2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}\log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+t}{\cot\left(\frac{\alpha}{2}\right)-t}\right|+c$$

$$=\cos ec\alpha\log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+\tan\left(\frac{x}{2}\right)}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{x}{2}\right)}\right|+c$$

$$=\cos ec\alpha\log\left|\frac{\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)\tan\left(\frac{x}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\tan\left(\frac{x}{2}\right)}\right|+c$$

4) Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)}dx$

Ans. : Let $I=\int \frac{\sin(x-a)}{\sin(x+a)}dx$

$$=\int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$$

$$=\int\left[\frac{\sin(x+a)\cos 2a-\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$$

$$=\int[\cos 2a-\sin 2a\cot(x+a)]dx$$

$$=x\cos 2a-\sin 2a\log|\sin(x+a)|+c$$

Section - C (6 Marks)

5) Evaluate $\int x\sec^2 x\,dx$

Ans. : Let $I=\int x\sec^2 x\,dx$

$$=x\int \sec^2 x\,dx-\int\left[\frac{d}{dx}(x)\int \sec^2 x dx\right]dx$$

$$=x\tan x-\int(1.\tan x)dx$$

$$=x\tan x-\int \tan x\,dx$$

$= x\tan x - \log|\sec x| + c$

$\therefore \quad I = x\tan x - \log|\sec x| + c$

OR

Evaluate $\displaystyle\int \frac{2x^2-1}{(x^2+4)(x^2+5)}\,dx$

Ans. : Let $I = \displaystyle\int \frac{2x^2-1}{(x^2+4)(x^2+5)}\,dx$

put $x^2 = t$.

$$\therefore \quad \frac{2x^2-1}{(x^2+4)(x^2+5)} = \frac{2t-1}{(t+4)(t+5)}$$

$$= \frac{A}{t+4} + \frac{B}{t+5}$$

$\therefore \quad 2t - 1 = A(t+5) + B(t+4)$

Put $t = -4$, $2(-4) - 1 = A(1) \quad \therefore A = -9$

Put $t = -5$, $2(-5) - 1 = B(-1) \therefore B = 11$

$$\therefore \quad \frac{2t-1}{(t+4)(t+5)} = \frac{-9}{t+4} + \frac{11}{t+5}$$

$$\therefore \quad I = \int\left(\frac{-9}{x^2+4} + \frac{11}{x^2+5}\right)dx$$

$$= -9\int \frac{1}{x^2+2^2}\,dx + 11\int \frac{1}{x^2+\left(\sqrt{5}\right)^2}\,dx$$

$$= \frac{-9}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$$

6) Evaluate $\displaystyle\int \frac{4\sin x - 38\cos x}{5\sin x - 11\cos x}dx$

Ans. : Let $I = \displaystyle\int \frac{4\sin x - 38\cos x}{5\sin x - 11\cos x}dx$

$\therefore\ 4\sin x - 38\cos x = A(5\sin x - 11\cos x)$

$+B\dfrac{d}{dx}(5\sin x - 11\cos x)$

$= A(5\sin x - 11\cos x) + B(5\cos x + 11\sin x)$

$= 5A\sin x - 11A\cos x + 5B\cos x + 11B\sin x$

$\therefore\ 4\sin x - 38\cos x = (5A + 11B)\sin x + (-11A + 5B)\cos x$

$5A + 11B = 4$ .... (i)

$-11A + 5B = -38$ .... (ii)

solving (i) and (ii), we get

$\therefore\ A = 3,\ B = 1$

$\therefore\ 4\sin x - 38\cos x = 3(5\sin x - 11\cos x) + 1(5\cos x + 11\sin x)$

$$\therefore I = \int\left[\frac{3(5\sin x - 11\cos x) + 1(5\cos x + 11\sin x)}{5\sin x - 11\cos x}\right]dx$$

$$= \int 3 + \frac{1(5\cos x + 11\sin x)}{5\sin x - 11\cos x}dx$$

$$= \int 3dx + \int \frac{(5\cos x + 11\sin x)}{5\sin x - 11\cos x}dx$$

$$= 3x + \log\left|5\sin x - 11\cos x\right| + c$$

Section - D (8 Marks)

7) If u and v are function of x then prove that

$$\int uvdx = u\int v\,dx - \int\left(\frac{du}{dx}\int vdx\right)dx$$

Ans. : Let $\int vdx = w \quad \therefore \dfrac{dw}{dx} = v$

Now $\dfrac{d}{dx}(uw) = u\dfrac{dw}{dx} + w\dfrac{du}{dx}$

$\dfrac{d}{dx}(uw) = uv + \int vdx\dfrac{du}{dx}$

By defination of integration

$$uw = \int\left[uv + \int vdx\frac{du}{dx}\right]dx$$

$$u\int vdx = \int uvdx + \int\left[\int vdx\frac{du}{dx}\right]dx$$

$$\therefore \quad \int uvdx = u\int vdx - \int\left[\frac{du}{dx}\int vdx\right]dx$$

8) Prove that :

$$\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

Ans. : Let $I = \int\sqrt{a^2-x^2}\cdot 1\,dx$

$$\therefore \ I = \sqrt{a^2-x^2}\int dx - \int\left[\frac{d}{dx}\sqrt{a^2-x^2}\int dx\right]dx$$

$$= x\sqrt{a^2-x^2} - \int\left[\frac{1}{2\sqrt{a^2-x^2}}.(-2x).x\right]dx$$

$$= x\sqrt{a^2 - x^2} - \int \left[\frac{-x^2}{\sqrt{a^2 - x^2}}\right] dx$$

$$= x\sqrt{a^2 - x^2} - \int \left[\frac{-x^2 + a^2 - a^2}{\sqrt{a^2 - x^2}}\right] dx$$

$$= x\sqrt{a^2 - x^2} - \int \left[\frac{a^2 - x^2 - a^2}{\sqrt{a^2 - x^2}}\right] dx$$

$$= x\sqrt{a^2 - x^2} - \int \left[\frac{a^2 - x^2}{\sqrt{a^2 - x^2}} - \frac{a^2}{\sqrt{a^2 - x^2}}\right] dx$$

$$= x\sqrt{a^2 - x^2} - \int \frac{a^2 - x^2}{\sqrt{a^2 - x^2}} dx$$

$$+ \int \frac{a^2}{\sqrt{a^2 - x^2}} dx$$

$$= x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2} dx$$

$$+ a^2 \int \frac{1}{\sqrt{a^2 - x^2}} dx$$

$$\therefore \quad I = x\sqrt{a^2 - x^2} - I + a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\therefore \quad 2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\therefore \quad I = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

OR

Evaluate : $\displaystyle\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$

Ans. : $$\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = \int \frac{1 + \cos 4x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{2\cos^2 2x.\sin x.\cos x}{\cos^2 x - \sin^2 x} dx$$

$$= \int \frac{\cos^2 2x.2\sin x.\cos x}{\cos 2x} dx$$

$$= \int \cos 2x \sin 2x \, dx$$

$$= \frac{1}{2}\int 2\sin 2x \cos 2x \, dx$$

$$= \frac{1}{2}\int \sin 4x \, dx$$

$$= \frac{1}{2}\frac{(-\cos 4x)}{4} + c$$

$$= -\frac{1}{8}\cos 4x + c$$

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