SHIKSHA CLASSES

Subject : Math -II **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 2. Application of Derivative**

Section - A (2 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) A rectangle has an area 25 sq. cm. If its perimerer is least its dimensions are

Ans. : a) 5,5

Hint: Let length = x , breadth = y, A = xy

$\therefore\ xy = 25 \qquad \therefore\ y = \frac{25}{x}$

$P = 2(x+y) \quad \therefore\ P = 2\left(x + \frac{25}{x}\right)$

$\therefore\ P = 2x + \frac{50}{x}$

$\therefore\ \frac{dP}{dx} = 2 - \frac{50}{x^2} \qquad \therefore\ \frac{d^2P}{dx^2} = \frac{100}{x^3}$

$\frac{dP}{dx} = 0 \qquad \therefore\ 2 - \frac{50}{x^2} = 0$

$\therefore\ 2 = \frac{50}{x^2} \qquad \therefore\ x^2 = 25$

$\therefore\ x = 5$ At $x = 5$, $\quad \therefore\ \frac{d^2P}{dx^2} = \frac{100}{125} > 0$

Product is maximum at $x = 5$

when $x = 5$, $y = \frac{25}{5} = 5$

2) The volume of a ball is increasing at the rate of $4\pi cm^3$ / sec. Find rate of increase of the radius when the volume is 288π cm^3

Ans. : b) $\frac{1}{36} cm/\sec$

$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\cancel{4}\cancel{\pi} = \cancel{4}\cancel{\pi} r^2 \frac{dr}{dt}$

$1 = r^2 \frac{dr}{dt}$

$\frac{1}{r^2} = \frac{dr}{dt}$

& $v \Rightarrow \frac{4}{3}\cancel{\pi} r^3 = 288\cancel{\pi}$

$\Rightarrow r^3 = \frac{\overset{72}{\cancel{288}} \times 3}{\cancel{4}_1}$

$r^3 = 216$

$r^3 = 6^3$

$r = 6$

$\therefore \frac{dr}{dt} = \frac{1}{r^2} = \frac{1}{(6)^2}$

$\therefore \frac{dr}{dt} = \frac{1}{36} cm/\sec$

Section - B (4 Marks)

3) Find the local maximum and local minimum value of $f(x) = x^3 - 3x^2 - 24x + 5$.

Ans. : Given that

$$f(x) = x^3 - 3x^2 - 24x + 5 \quad \text{---(i)}$$

Differentiate (i) w.r.t.x.

$f'(x) = 3x^2 - 6x - 24$ ---(ii)

For extreme values, $f'(x) = 0$

$3x^2 - 6x - 24$ i.e. $3(x^2 - 2x - 8) = 0$

i.e. $x^2 - 2x - 8 = 0$ i.e.

$(x + 2)(x - 4) = 0$

$\Rightarrow x + 2 = 0$ or $x - 4 = 0 \Rightarrow x = -2$ and $x = 4$.

The stationary points are $x = -2$ and $x = 4$.

Differentiate (ii) w.r.t.x.

$f''(x) = 6x - 6$ ---(iii)

For $x = -2$, from (iii) we get,

$f''(-2) = 6(-2) - 6 = -18 < 0$

$\therefore$ At $x = -2$, f (x) has a maximum value.

For maximum of f (x), put $x = -2$ in (i)

$f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 5 = 33$.

For $x = 4$, from (iii) we get

$f''(4) = 6(4) - 6 = 18 > 0$

$\therefore$ At $x = 4$, f (x) has a minimum value.

$f(4) = (4)^3 - 3(4)^2 - 24(4) + 5 = -75$

$\therefore$ Local maximum of f (x) is 33 when $x = -2$ and Local minimum of f (x) is -75 when $x = 4$.

OR

A wire of length 36 meters is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.

Ans. : Let x m and y m be the length and bredth of the rectangle.

Then its perimeter is $2(x + y) = 36$

$\therefore x + y = 18 \quad \therefore y = 18 - x$

Area of the rectangle $= xy = x(18 - x)$

Let $f(x) = x(18 - x) = 18x - x^2$

$\therefore f'(x) = \dfrac{d}{dx}(18 - x^2) = 18 - 2x$

and $f''(x) = \dfrac{d}{dx}(18 - 2x) = 0 - 2 \times 1 = -2$

Now $f'(x) = 0$, if $18 - 2x = 0$

i.e., if $x = 9$

and $f''(9) = -2 < 0$

$\therefore$ by the second derivative test f has maximum value at $x = 9$.

When $x = 9$, $y = 18 - = 9$

$\therefore x = 9$ m, $y = 9$ m

$\therefore$ the rectangle is a square of side 9 m.

4) Find the equation of tangent to the curve $2x^2 + 3y^2 - 5 = 0$ at (1,1)

Ans. : $2x^2 + 3y^2 - 5 = 0$

Differentiate w.r.t.x ,we get

$$4x + 6y\frac{dy}{dx} = 0 \ \backslash 6y\frac{dy}{dx} = -4x$$

$$\therefore \frac{dy}{dx} = -\frac{4x}{6y} \qquad \therefore \frac{dy}{dx} = -\frac{2x}{3y}$$

$$\therefore \left(\frac{dy}{dx}\right)_{at(1,1)} = -\frac{2(1)}{3(1)} = -\frac{2}{3}$$

$\therefore$ Slope of tangent $= -\dfrac{2}{3}$

$\therefore$ Equation of tangent is

$y - y_1 = m(x - x_1)$

$\therefore y - 1 = -\dfrac{2}{3}(x - 1)$

$\therefore 3y - 3 = -2x + 2$

$\therefore 2x + 3y - 5 = 0$

$\therefore$ Equation of tangent $2x + 3y - 5 = 0$

Section - C (6 Marks)

5) Divide the number 70 in two parts so that the their product is maximum.

Ans. : Let the first part be x

$\therefore$ The second part be $70 - x$

$P = x(70 - x)$

$= 70x - x^2$

$f(x) = 70x - x^2$

$\therefore f'(x) = 70 - 2x \qquad \therefore f''(x) = -2$

$f'(x) = 0$

$\therefore 70 - 2x = 0 \qquad \therefore 2x = 70$

$\therefore x = 35$

$\therefore f''(35) = -2 < 0$

$\therefore$ Product is maximum at $x = 35$

$\therefore$ First part = 35

$\therefore$ Second part = 70 – 35 = 35

$\therefore$ The two parts are 35, 35

OR

Determine the maximum and minimum value of $f(x) = 2x^3 - 21x^2 + 36x - 20$

Ans. : $f(x) = 2x^3 - 21x^2 + 36x - 20$

$\therefore\ f'(x) = 6x^2 - 42x + 36$

$\therefore\ f'(x) = 12x - 42$

$f'(x) = 0$

$\therefore\ 6x^2 - 42x + 36 = 0$

$\therefore\ x^2 - 7x + 6 = 0$

$\therefore\ (x - 6)(x - 1) = 0$

$\therefore\ x = 6, x = 1$

when $x = 6$

$f''(x) = f''(6) = 12(6) - 42 = 72 - 42 = 30 > 0$

$\therefore\ f'(x) > 0$

$\therefore$ f has minima at $x = 6$

$f(\min) = 2(6)^3 - 21(6)^2 + 36(6) - 20$

$= 432 - 756 + 216 - 20$

$= -128$

$\therefore$ Minimum value $= -128$ at $x = 6$

Now when $x = 1$

$f''(x) = 12(1) - 42 = 12 - 42 = -30 < 0$

$\therefore\ f'(x) < 0$

$\therefore$ f has maxima at $x = 1$.

$f(max) = 2(1)^3 - 21(1)^2 + 36(1) - 20$

$= 2 - 21 + 36 - 20$

$= -3$

$\therefore$ Maximum value $= -3$ at $x = 1$

6) The radius of a circle is increasing at the rate of 2 cm /sec. Find the rate at which the area of the circle is increasing, when the radius is 5 cm.

Ans. : Let radius of circle = x cm given

$x = 5$ cm, $\frac{dx}{dt} = 2$

$\therefore\ A = \pi x^2$

$\therefore\ \frac{dA}{dt} = 2\pi x.\frac{dx}{dt}$

$= 2 \times \pi \times 5 \times 2 = 20\pi$ cm²/ sec.

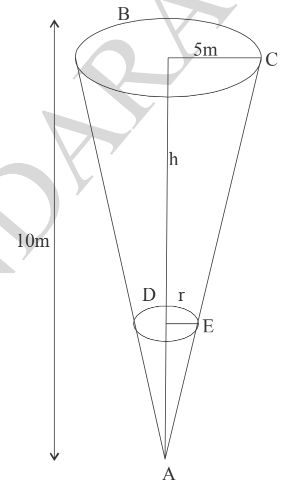
$\therefore$ Area is increasing at the rate of 20π cm²/sec.

Section - D (8 Marks)

7) An inverted cone of 10m height and 5m base radius is filled with water. Its volums increase at the rate of $\frac{3\pi}{2} CC/\min$. find the rate of which the height of water level when the depth is 4m.

Ans. : Let r be the radius and h be the height of the watcrcone formed at the time t.

From the fig,



$\Delta ABC \sim \Delta ADE$

$\frac{BC}{DE} = \frac{AB}{AD}$

$\frac{5}{r} = \frac{10}{h}$

$\therefore\ r = \frac{5h}{10}$

$\therefore\ \boxed{r = \frac{h}{2}}$

Let v be the volume of water - cone at time t.

$\therefore \frac{dv}{dt} = \frac{3}{2} CC/\min$

$v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 = \frac{1}{3}\pi\frac{h^3}{4} =$

$= \frac{1}{12}\pi h^3$

$\frac{dv}{dt} = \frac{1}{12}\pi 3h^2 \frac{dh}{dt}$

$\frac{3}{2}\cancel{\pi} = \frac{1}{4}\cancel{\pi} h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{\cancel{2}_1} \times \frac{\cancel{4}^2}{h^2}$

$\frac{dh}{dt} = \frac{6}{h^2}$

$\frac{dh}{dt} = \frac{6}{16} = \frac{3}{8}$

$\therefore$ The level of water is rising at the rate of $\frac{3}{8}$ m/min.

8) A stone, vertically thrown upwards is moving in a live. Its equation of motion is

$S = 294t - 49t^2$ then find the maximum height that the stone reaches.

Ans. : we have

$S = 294t - 49t^2$ ---(1)

then $v = \frac{ds}{dt} = 294 - 98t$

But when V = 0

$\therefore\ 294 - 98t = 0$

$-98t = -294$

$98t = 294$

$t = \frac{294^{3}}{98_{1}}$

$t = 3\,\text{sec}$

$\therefore$ from eq^n (1),

$\therefore S = 294 \times 3 - 49 \times (3)^2$

$= 882 - 441$

$S = 441$

Hence Height of stone reaches is 441 units.

OR

A manufacturer can sell x items at the rate of Rs.(330 – x) each. The cost of producing x items is Rs.$x^2 + 10x + 12$ How many items must be sold so that his profit is maximum?

Ans. : Let the number of items = x

S.P of one items = ₹ $(330 - x)$

$\therefore$ S.P of x items = ₹ $(330 - x)x$

C.P of x items = ₹ $(x^2 + 10x + 12)$

$\therefore$ profit = S.P – C.P

$= (330 - x)x - (x^2 + 10x + 12)$

$= 330x - x^2 - x^2 - 10x - 12$

$= -2x^2 + 320x - 12$

Let $f(x) = -2x^2 + 320x - 12$

$\therefore\ f'(x) = -4x + 320 \quad \therefore\ f''(x) = -4$

$f'(x) = 0$

$-4x + 320 = 0$

$\therefore\ 4x = 320 \quad \therefore\ x = 80$

$f''(80) = -4 \quad \therefore\ f''(80) < 0$

$\therefore\ f$ has maximum value at $x = 80$

Hence the profit is maximum at $x = 80$

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