SHIKSHA CLASSES

Subject : Math -II **BOARD ANSWER PAPER** **Total Marks : 20**
Class : XII **Topic: 1. Differentiation**

Section - A (2 Marks)

Select and write the correct answer from the given alternatives for each of the following :

1) If y = e^x + log x then $\frac{dy}{dx}$ at x = 1 is

Ans. d) 1 + e

$$y = e^x + \log x$$

$$\frac{dy}{dx} = e^x + \frac{1}{x}$$

put $x = 1$

$$\therefore \frac{dy}{dx} = e^1 + \frac{1}{1} = e + 1$$

2) $\frac{d}{dx}\tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right) =$

Ans.: d) $\frac{3}{1+(3x+2)^2} + \frac{2}{1+(2x-1)^2}$

$$y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$$

$$\tan^{-1}\left(\frac{A+B}{1-AB}\right) = \tan^{-1} A + \tan^1 B$$

$$\tan^{-1}\left(\frac{(3x+2)+(2x-1)}{1-(3x+2)(2x-1)}\right)$$

$$\therefore y = \tan^1(3x+2) + tam^{-1}(2x-1)$$

diff. w.r.t.x.

$$\frac{dy}{dx} = \frac{1}{1+(3x+2)^2}\frac{d}{dx}(3x+2)$$

$$+ \frac{1}{1+(2x-1)^2}\frac{d}{dx}(2x-1)$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+(3x+2)^2} \times (3\times1+0)$$

$$+ \frac{1}{1+(2x-1)^2} \times (2\times1-0)$$

$$\therefore \frac{dy}{dx} = \frac{3}{1+(3x+2)^2} + \frac{2}{1+(2x-1)^2}$$

Section - B (4 Marks)

3) Find $\frac{dy}{dx}$ if $y = x^x$

Ans. : $y = x^x$

Taking log on both side

$$\log y = \log x^x$$

$$\therefore \log y = x \log x$$

Differentate *w.r.t. t*, we get,

$$\frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + \log x.1$$

$$\therefore \frac{dy}{dx} = y(1 + \log x)$$

$$\therefore \frac{dy}{dx} = x^x(1 + \log x)$$

OR

If $y = e^{m\cos^{-1}x}$ then show that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$$

Ans. : Let y = $e^{m\cos^{-1}x}$

Differentiate w.r.t.x ,we get

$$\therefore \quad \frac{dy}{dx} = e^{m\cos^{-1} x}.m.\frac{-1}{\sqrt{1-x^2}}$$

$$\therefore \quad \sqrt{1-x^2}\,\frac{dy}{dx} = -my$$

squaring on both side ,we get

$$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

Again differentiate w.r.t.x ,we get

$$\therefore \left(1-x^2\right).2\frac{dy}{dx}.\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = m^2.2y.\frac{dy}{dx}$$

$$\therefore \quad \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

... (dividing by $2\frac{dy}{dx}$)

$$\therefore \quad \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

Hence proved.

4) Find $\frac{dy}{dx}$, if x = at² and y = 2at

Ans. : Ans : $x = at^2, y = 2at$

Differentate w.r.t. t, we get,

$$\therefore \quad \frac{dx}{dt} = 2at \qquad \therefore \quad \frac{dy}{dt} = 2a$$

$$\therefore \quad \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$$

$$= \frac{2a}{2at} = \frac{1}{t} \qquad \therefore \quad \frac{dy}{dx} = \frac{1}{t}$$

Section - C (6 Marks)

5) Find $\frac{dy}{dx}$, if y = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Ans. : $$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

put $x = \tan\theta \quad \therefore \theta = \tan^{-1} x$

$$\therefore \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}\left(\sin 2\theta\right) = 2\theta$$

$$\therefore \quad y = 2\tan^{-1} x$$

$$\therefore \quad \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

OR

If $x = e^{\sin 3t}$, $y = e^{\cos 3t}$ show that

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y}$$

Ans. : $x = e^{\sin 3t}, \quad y = e^{\cos 3t}$

Taking log on both side , we get

$$\log x = \log e^{\sin 3t}, \ \log y = \log e^{\cos 3t}$$

$$\log x = \sin 3t \log e, \ \log y = \cos 3t \log e$$

$$\log x = \sin 3t, \ \log y = \cos 3t$$

Differentiate w.r.t.x , we get

$$\frac{1}{x}\frac{dx}{dt} = 3\cos 3t, \ \frac{1}{y}\frac{dy}{dt} = -3\sin 3t$$

$$\frac{dx}{dt} = 3x\cos 3t, \ \frac{dy}{dt} = -3y\sin 3t$$

$$\therefore \quad \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$$

$$\therefore \quad \frac{dy}{dx} = \frac{-3y\sin 3t}{3x\cos 3t}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{y\log x}{x\log y}$$

6) $y = \log\left[\frac{x+\sqrt{x^2+25}}{\sqrt{x^2+25}-x}\right]$ **find** $\frac{dy}{dx}$

Ans. :

Let $y = \log\left[\frac{x+\sqrt{x^2+25}}{\sqrt{x^2+25}-x}\right]$

$$y = \log\left(x+\sqrt{x^2+25}\right) - \log\left(\sqrt{x^2+25}-x\right)$$

Differentiate w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+25}}\frac{d}{dx}\left(x+\sqrt{x^2+25}\right)$$

$$-\frac{1}{\sqrt{x^2+25}-x}\frac{d}{dx}\left(\sqrt{x^2+25}-x\right)$$

$$=\frac{1}{x+\sqrt{x^2+25}}\left[1+\frac{1}{2\sqrt{x^2+25}}\frac{d}{dx}\left(x^2+25\right)\right]$$

$$-\frac{1}{\sqrt{x^2+25}-x}\left[\frac{1}{2\sqrt{x^2+25}}\frac{d}{dx}\left(x^2+25\right)-1\right]$$

$$=\frac{1}{x+\sqrt{x^2+25}}\left(1+\frac{1}{2\sqrt{x^2+25}}.(2x)\right)$$

$$-\frac{1}{\sqrt{x^2+25}-x}\left(\frac{1}{2\sqrt{x^2+25}}.(2x)-1\right)$$

$$=\frac{1}{x+\sqrt{x^2+25}}\left(1+\frac{x}{\sqrt{x^2+25}}\right)$$

$$-\frac{1}{\sqrt{x^2+25}-x}\left(\frac{x}{\sqrt{x^2+25}}-1\right)$$

$$=\frac{1}{x+\sqrt{x^2+25}}\left(\frac{\sqrt{x^2+25}+x}{\sqrt{x^2+25}}\right)$$

$$-\frac{1}{\sqrt{x^2+25}-x}\left(\frac{x-\sqrt{x^2+25}}{\sqrt{x^2+25}}\right)$$

$$=\frac{1}{x+\sqrt{x^2+25}}\left(\frac{\sqrt{x^2+25}+x}{\sqrt{x^2+25}}\right)$$

$$+\frac{1}{\sqrt{x^2+25}-x}\left(\frac{\sqrt{x^2+25}-x}{\sqrt{x^2+25}}\right)$$

$$=\frac{1}{\sqrt{x^2+25}}+\frac{1}{\sqrt{x^2+25}}=\frac{2}{\sqrt{x^2+25}}$$

$$\therefore \quad \frac{dy}{dx}=\frac{2}{\sqrt{x^2+25}}$$

Section - D (8 Marks)

7) If y is a differentiable function of u and u is a differentiable function of x, then prove that

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$$

Ans. : Let δu and δy be the increments in u and y respectively, corresponding to the increment δx in x.

Now y is a differentiable function of u and u is a differentiable function of x.

$$\therefore \quad \frac{dy}{du}=\lim_{\delta u\to 0}\frac{\delta y}{\delta u} \text{ and } \frac{du}{dx}=\lim_{\delta x\to 0}\frac{\delta u}{\delta x}$$

Now, $\frac{\delta y}{\delta x}=\frac{\delta y}{\delta u}\times\frac{\delta u}{\delta x}$

Taking limits as $\delta x\to 0$, we get

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\left(\frac{\delta y}{\delta u}\times\frac{\delta u}{\delta x}\right)$$

$$\therefore \quad \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\frac{\delta y}{\delta u}\times\lim_{\delta x\to 0}\frac{\delta u}{\delta x}$$

as $\delta x\to 0,\ \partial u\to 0$

$$\therefore \quad \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta u\to 0}\frac{\delta y}{\delta u}\times\lim_{\delta x\to 0}\frac{\delta u}{\delta x}$$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$$ Hence proved.

8) If $x^p y^q=(x+y)^{p+q}$ show that $\frac{d^2y}{dx^2}=0$

Ans. :

$x^p.y^q=(x+y)^{p+q}$

Taking log on both side , we get

$$\therefore \quad \log\left(x^p.y^q\right)=\log\left(x+y\right)^{p+q}$$

$$\therefore \quad \log x^p+\log y^q=\log\left(x+y\right)^{p+q}$$

$$\therefore \quad p\log x+q\log y=(p+q)\log\left(x+y\right)$$

Differentiate w.r.t x, we get

$\therefore$

$$p.\frac{1}{x}+q.\frac{1}{y}\frac{dy}{dx}=(p+q).\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\therefore \quad \frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\therefore \quad \frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}+\frac{p+q}{x+y}\frac{dy}{dx}$$

$$\therefore \quad \frac{q}{y}\frac{dy}{dx}-\frac{p+q}{x+y}\frac{dy}{dx}=\frac{p+q}{x+y}-\frac{p}{x}$$

$$\therefore \quad \left(\frac{q}{y}-\frac{p+q}{x+y}\right)\frac{dy}{dx}=\frac{p+q}{x+y}-\frac{p}{x}$$

$$\therefore \left(\frac{qx+qy-py-qy}{y(x+y)}\right)\frac{dy}{dx}=\frac{px+qx-px-py}{x(x+y)}$$

$$\therefore \quad \left(\frac{qx-py}{y(x+y)}\right)\frac{dy}{dx}=\frac{qx-py}{x(x+y)}$$

$$\therefore \quad \frac{dy}{dx}=\frac{qx-py}{x(x+y)}\times\frac{y(x+y)}{qx-py}$$

$$\therefore \quad \frac{dy}{dx}=\frac{y}{x}$$

Again differentiate w.r.t.x , we get

$$\therefore \frac{d^2y}{dx^2}=\frac{x\frac{dy}{dx}-y.1}{x^2}=\frac{x\left(\frac{y}{x}\right)-y.1}{x^2}=\frac{y-y}{x^2}=0$$

$$\therefore \frac{d^2y}{dx^2}=0$$

Hence proved.

OR

If $x=\sin t$ $y=e^{mt}$ then show that

$$\mathbf{(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0.}$$

Ans. : Given that x= sin t

$\therefore \quad t=\sin^{-1}x$

and $y=e^{mt}$

$\therefore \quad y=e^{m\sin^{-1}x}$ ---(i)

Differentiate w.r.t.x

$$\frac{dy}{dx}=\frac{d}{dx}(e^{m\sin^{-1}x})=e^{m\sin^{-1}x}.\,m\frac{d}{dx}(\sin^{-1}x)$$

$$\frac{dy}{dx}=\frac{m.e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx}=my \qquad \text{---[From(i)]}$$

Squaring both sides

$$(1-x^2).\left(\frac{dy}{dx}\right)^2=m^2y^2$$

Diferentiate w.r.t.x

$$(1-x^2)\frac{d}{dx}\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)^2\frac{d}{dx}(1-x^2)=m^2\frac{d}{dx}(y^2)$$

$$(1-x^2).2\left(\frac{dy}{dx}\right).\frac{d}{dx}.\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2(-2x)=m^2(2y)\frac{dy}{dx}$$

$$2(1-x^2).\frac{dy}{dx}.\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=2m^2y\frac{dy}{dx}$$

Dividing throughout by $2\frac{dy}{dx}$ we get,

$$(1-x^2).\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y$$

$$\therefore \quad (1-x^2).\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y=0$$

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